

ONE POSSIBLE INTERACTION-INERTIAL INTERACTION

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Abstract

Proposed in this paper is a possible interaction which exists in nature - inertial interaction. It gives matter an inertia and inertial mass. The formula of inertial mass has been derived. It is possible that inertial interaction leads to the redshifts of quasars, the rotation curve of spiral galaxy, the accelerating expansion of the universe, and the stronger gravitational lens effects of quasars, galaxies, or clusters of galaxies. Einstein's Gravitational Equation has been modified. Gravitational redshift, perihelion precession, and bending of light in spherically symmetric vacuum gravitational field are calculated. The differential equations of static spherically symmetric star's internal evolution are given. The accelerating expansion stage of the universe evolution equations are derived. The evolution of the universe is periodic. Time does not have an origin. There is no Big Bang. Although there is divergent singularity, there is no universe's singularity of incomplete geodesic. There are no horizon problem and no flatness problem. The problems that may exist are discussed.

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1 INTRODUCTION

It is predicted that perhaps there exists inertial interaction in nature and the inertial interaction gives matter an inertia and inertial mass. It is possible that inertial interaction leads to the redshifts of quasars, the rotation curve of spiral galaxy, the accelerating expansion of the universe, and the stronger gravitational lens effects of quasars, galaxies, or clusters of galaxies.

In Newton's bucket experiment[1], how can the water know that only when it rotates relative to the distant galaxies in the universe, instead of relative to the bucket wall, the concavity of the water surface changes from convex to concave? It can be assumed that the concave water surface is the result of the interaction of distant galaxies in the universe to the water; or, the inertia of matter is the result of the interaction of other substances to the matter. Let us imagine that the entire universe's substance is electrically neutral and distributed as a uniform spherical shell. There is an object in the universe's spherical shell. The inertia of the object is the result of the interaction of spherical shell universe's substance to the object itself. The interaction of the object is obviously not from the four fundamental interactions of the spherical shell universe. A possible alternative is that, let us imagine, this kind of interaction is a new interaction which is different from any one of the four fundamental interactions. Or, at least, it is another aspect of the gravitational interaction which people have not recognized. If such an interaction exists, we can call it inertial interaction or gravitational inertial interaction.

2 THE FORMULA OF INERTIAL MASS

2.1 THE CONTRIBUTION OF INERTIAL MASS FROM COSMIC BACKGROUND

Inertial interaction gives matter an inertia and inertial mass. Let M_{I12} and M_{I21} be respectively the inertial masses of particle 1 to particle 2 and particle 2 to particle

1. Assume

$$M_I \equiv M_{I12} = M_{I21} = K \frac{M_{G1}M_{G2}}{r^n} \exp(-\delta r). \quad (1)$$

where, M_{G1} and M_{G2} are the gravitational masses of particle 1 and particle 2 respectively. r is the length of the geodesic between particle 1 and particle 2. $K(> 0)$ and $\delta(> 0)$ are the constants to be determined. n is an integer to be determined. If the inertial interaction is weaker than the gravitational interaction and if it is long-range, K and δ will be very small.

The following calculation is for the inertial mass of the particle, which is the result of the inertial interaction of the cosmic background to the particle with gravitational mass M_G . Let us assume that the cosmological principle holds, then the universe metric is Robertson-Walker metric[2].

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right]. \quad (2)$$

Let us consider a flat space($k = 0$) (Other cases can also be calculated in the same manner.). The distance between the two points $A(r_A, \theta, \varphi)$ and $B(r_B, \theta, \varphi)$ is[2]

$$D_{AB}(t) = a(t) \int_{r_A}^{r_B} dr / \sqrt{1 - kr^2} = a(t)(r_B - r_A). \quad (3)$$

Let the origin point of Robertson-Walker coordinate be the point where the particle sits, and the volume element of the point (r, θ, φ) is[2]

$$\hat{\varepsilon} = a^3(t) r^2 \sin\theta dr \Lambda d\theta \Lambda d\varphi. \quad (4)$$

The inertial mass of the particle, which is the result of the inertial interaction of the cosmic background to the particle, is

$$\begin{aligned} M'_I(t) &= \int_{\Sigma} K \frac{M_G}{(ar)^n} \exp(-\delta ar) \rho_G \hat{\varepsilon} \\ &= K \frac{4\pi M_G \rho_G}{\delta^{3-n}} \Gamma(3 - n) \equiv \alpha_U M_I \end{aligned} \quad (5)$$

where, $\Gamma(3 - n)$ is Γ function. α_U is the contribution rate of cosmic background for inertial mass of the particle. Σ is the space of universe at moment t . ρ_G is the gravitational mass density of the universe at moment t . α_U is the function of time t through ρ_G . On the Earth today $M_I = M_G$, From equation (5), we can obtain

$$K \frac{4\pi\rho_G}{\alpha_U \delta^{3-n}} \Gamma(3-n) = 1. \quad (6)$$

The inertial mass of the particle on the Earth is mainly contributed by the cosmic background, the Milky Way, the Sun and the Earth's inertial interaction. Let α_U , α_M , α_\odot , and α_E be the contribution rates respectively. Today $\alpha_U + \alpha_M + \alpha_\odot + \alpha_E = 1$.

2.2 THE ROTATION CURVE OF SPIRAL GALAXY AND THE FORMULA OF INERTIAL MASS

The inertial mass of a star in a spiral galaxy is mainly contributed by the background of the universe and the spiral galaxy through inertial interaction. The inertial mass of the star is

$$M_{IS} = \alpha_U M_G + K \frac{M_{GSg} M_G}{(r_S)^n} \exp(-\delta r_S). \quad (7)$$

where, M_{GSg} is the gravitational mass of the luminous part of the spiral galaxy. r_S is the distance from the star to the center of the spiral galaxy. M_G is the gravitational mass of the star. If $n = 1$, $\delta r_S \ll 1$, and $\alpha_U \ll K \frac{M_{GSg}}{r_S}$, at the outside of the core of the spiral galaxy, from $M_{IS} \frac{V_0^2}{r} = G \frac{M_{GSg} M_G}{r^2}$ and equation (7)) we can obtain (Here, equation (23) is used)

$$V_0 = \sqrt{G \frac{M_{GSg} M_G}{r M_{IS}}} \simeq \sqrt{\frac{G}{K}}. \quad (8)$$

where, V_0 is the velocity of the star rotating around the core of the spiral galaxy. From equation (8) we know that V_0 is approximately a constant and V_0 is independent of the gravitational mass of the light-emitting part of the spiral galaxy. Almost all the V_0 of all spiral galaxies are the same. The formula (1) of inertial mass between particle 1 and particle 2 becomes

$$M_I \equiv M_{I12} = M_{I21} = K \frac{M_{G1} M_{G2}}{r} \exp(-\delta r). \quad (9)$$

where K can be obtained from equation (8)

$$K = \frac{G}{V_0^2}. \quad (10)$$

Let $G = 6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2$ and $V_0 = 2 \times 10^5 \text{m/s}$, then $K = 1.67 \times 10^{-21} \text{m/kg}$.

From calculation, we know that the inertial mass of a star at the center of spiral galaxy is $M_{IS} = \frac{3}{2}K \frac{M_{GSg}M_G}{r_0}$. We can also know that the inertial mass of the star at the edge of spiral galaxy core is $M_{IS} \simeq \frac{3}{4}K \frac{M_{GSg}M_G}{r_0}$, where r_0 is the radius of the spiral galaxy core. Therefore, it can be approximately regarded that the inertial mass of a star in the spiral galaxy nucleus has nothing to do with the location within spiral galaxy nucleus of the star. From $M_{IS} \frac{V_0^2}{r} = G \frac{M_{GSr}M_G}{r^2}$, we have the speed of rotation of a star in the spiral galaxy nucleus

$$V_0 = \sqrt{G \frac{M_{GSr}M_G}{rM_{IS}}} \simeq \sqrt{G \frac{4\pi\rho_G M_G}{3M_{IS}}} r \propto r. \quad (11)$$

where, M_{GSr} is the gravitational mass of the spiral galaxy nucleus in the range of radius r . ρ_G is the gravitational mass density of the spiral galaxy nucleus.

In addition, $\frac{1}{M_I} \frac{dM_I}{dr} = -\frac{1}{r} - \delta$ can be obtained by using the formula (9) of inertial mass. On the scale of galaxy clusters, the inertial mass M'_I of the gas molecules within the cluster of galaxies almost is $M'_I \simeq \alpha_U M_I \approx 0.09 M_I$. M_I is the inertial mass of the same molecule on the Earth. The value of α_U can be found in §2.3 at the latter part. On one hand, by the following equation (30) we know that as a result of the gravitation the escape deceleration $\frac{d\mathbf{V}}{dt} = \frac{1}{M'_I} \mathbf{F} - \frac{1}{M'_I} \mathbf{V} \frac{dM'_I}{dt} \approx \frac{1}{\alpha_U M_I} \mathbf{F} \sim \frac{1}{0.09} \frac{1}{M_I} \mathbf{F}$ of the hot gases in galaxy clusters should be larger than we had expected. On the other hand, $\frac{1}{2} \times 0.09 M_I \bar{v}^2 \sim \frac{3}{2} K_B T$ and K_B is the Boltzmann constant. The thermal motion velocity $\sqrt{v^2} \sim \frac{1}{\sqrt{0.09}} \sqrt{\frac{3}{M_I} K_B T}$ of the molecular is also larger than we had expected. However, it is not larger than the escape deceleration which is larger than we had expected.

2.3 QUASAR REDSHIFT

The inertial mass of a particle on the surface of a star in or near the Milky Way is mainly contributed by the cosmic background, the Milky Way, and the star through the inertia interaction. The inertial mass of the particle is

$$M'_I = \alpha_U M_G + K \frac{M_{GM}M_G}{r_M} + K \frac{M_{GS}M_G}{R_S} = (\alpha_U + K \frac{M_{GM}}{r_M} + K \frac{M_{GS}}{R_S}) M_I. \quad (12)$$

where, M_{GM} is the gravitational mass of the Milky Way, r_M is the distance from the star to the galactic center, M_{GS} is the gravitational mass of the star, R_S is the radius of the star, M_G is the gravitational mass of the particle, $M_I (= M_G)$ is the inertial mass of the particle on the Earth. If $\alpha_U + K \frac{M_{GM}}{r_M} + K \frac{M_{GS}}{R_S} < 1$, the electron inertial mass on the surface of a star is less than the electron inertial mass on the Earth and the Rydberg constant on the surface of a star will be less than the Rydberg constant on the Earth, and then this will lead to the redshift of Stellar spectrum. For a star that is far from the galactic center, if $K \frac{M_{GS}}{R_S}$ is not big enough the spectrum possibly is redshift. If $K \frac{M_{GS}}{R_S}$ is big enough, resulting in $\alpha_U + K \frac{M_{GM}}{r_M} + K \frac{M_{GS}}{R_S} > 1$, then the spectrum will possibly be violet-shift. There are possible $\alpha_U + K \frac{M_{GM}}{r_M} + K \frac{M_{GS}}{R_S} < 1$ and larger redshifts for the stars, which are on the edge of the galaxy or beyond the Milky Way, and the stars are possible quasars. If $K \frac{M_{GM}}{r_M} \ll \alpha_U$ and $K \frac{M_{GS}}{R_S} \ll \alpha_U$ so that $M'_I \simeq \alpha_U M_I$, then the quasar redshift reaches maximum. Therefore, at least some of quasars are the stars that are on the edge or away from the Milky Way. Due to $M_{Iqe} \simeq (\alpha_U + \alpha_\odot) M_{Ie}$, the electron inertial mass on the surface of the quasar that has the maximum redshift (M_{Ie} is the electron inertial mass on the Earth) (here, as an approximation, α_\odot replaced $K \frac{M_{GS}}{R_S}$), the Rydberg constant on the surface of the quasar which has the maximum redshift is $R_q = (\alpha_U + \alpha_\odot) R$ (R is the Rydberg constant on the Earth). The wavelength of the spectrum emitted by the element on the surface of the quasar which has the maximum redshift is $\lambda_q = \frac{1}{(\alpha_U + \alpha_\odot)} \lambda$ (λ is the wavelength of the spectrum of the same elements on the Earth). The quasar maximum redshift is $Z_{qmax} \equiv \frac{\lambda_q - \lambda}{\lambda} = \frac{1}{(\alpha_U + \alpha_\odot)} - 1$, then $\alpha_U + \alpha_\odot = \frac{1}{Z_{qmax} + 1}$. Let the quasar maximum redshift be $Z_{qmax} = 5$, then $\alpha_U + \alpha_\odot = \frac{1}{6} \approx 0.17$ and $\alpha_U < 0.17$. On the Earth $\alpha_M \approx 10\alpha_\odot \approx 100\alpha_E$, then $\alpha_M \approx 0.82$ can be obtained from $\alpha_U + \alpha_M + \alpha_\odot + \alpha_E = 1$, and current $\alpha_U \approx 0.09$.

If a quasar is a distant active galaxy, all spectra of the quasar have cosmological redshift. If quasars are the stars that are on the edge of or away from the Milky Way, then only the emission lines and absorption lines of the quasar by the electronic transition in atom have redshifts and continuous spectrum have no redshift. If the X-

ray spectrum of a quasar is produced by bremsstrahlung and the radiation intensity is inversely proportional to the square of the inertial mass of the charged particle, then the radiation intensity of a quasar's X-ray will be approximately 10^2 times as big as that of ordinary stars. If the X-ray spectrum of a quasar is produced by synchrotron radiation and the radiation intensity is inversely proportional to 4 power of the inertial mass of the charged particle, then the radiation intensity of a quasar's X-ray will be approximately 10^4 times as big as that of ordinary stars.

From equation (5), we have $M'_I = K \frac{4\pi M_G \rho_G}{\delta^2} \equiv \alpha_U M_I = \alpha_U M_G$. Therefore, the cosmic gravitational mass density is

$$\rho_G(t) = \frac{\alpha_U \delta^2}{4\pi K} = \frac{\alpha_U \delta^2 V_0^2}{4\pi G}. \quad (13)$$

Due to $r_S \delta \ll 1$, let $\sigma r_S \delta \sim 1$, then

$$\rho_G(t) = \frac{\alpha_U V_0^2}{4\pi \sigma^2 r_S^2 G}. \quad (14)$$

Let $r_S \sim 10^{21}m$, $\alpha_U = 0.09$, $\sigma = 10^2$, and $V_0 = 2 \times 10^5 m/s$, then the universe current gravitational mass density is $\rho_G \sim 4.295 \times 10^{-28} kg/m^3$, and the universe current inertial mass density is $\rho_I = \alpha_U \rho_G \sim 3.866 \times 10^{-29} kg/m^3$.

2.4 THE CYCLE OF A PENDULUM AT PERIHELION AND APHELION

The formula (9) of inertial mass can be verified by measuring the cycle of a pendulum at Perihelion and aphelion. On the Earth, the cycle of a pendulum is:

$$T = 2\pi \sqrt{\frac{M_I}{M_G} \frac{l}{g}}. \quad (15)$$

where, M_I and M_G are the inertial mass and gravitational mass of the pendulum respectively. l is the length of the swing arm. g is the Earth's gravitational acceleration. The inertial mass of a body on the Earth is $M_I = (\alpha_\odot + \alpha_U + \alpha_M + \alpha_E)M_G \simeq (\frac{GM_\odot}{V_0^2 r} + \alpha_U + \alpha_M + \alpha_E)M_G$, then

$$T = 2\pi \sqrt{(\frac{GM_\odot}{V_0^2 r} + \alpha_U + \alpha_M + \alpha_E) \frac{l}{g}}. \quad (16)$$

The cycle of a pendulum at Aphelion is

$$T_A = 2\pi \sqrt{\left(\frac{GM_\odot}{V_0^2(r_A + R_E)} + \alpha_U + \alpha_M + \alpha_E\right) \frac{l}{g_A}}. \quad (17)$$

The cycle of a pendulum at Perihelion is

$$T_P = 2\pi \sqrt{\left(\frac{GM_\odot}{V_0^2(r_P - R_E)} + \alpha_U + \alpha_M + \alpha_E\right) \frac{l}{g_P}}. \quad (18)$$

where, r_A and r_P are respectively the aphelion distance and perihelion distance. g_A and g_P are the Earth's gravitational acceleration at the aphelion and perihelion respectively. R_E is the radius of the Earth. From calculation, we have

$$\frac{T_P - T_A}{T_A} \sim 10^{-4}. \quad (19)$$

3 THE MOTION EQUATION OF AN OBJECT

3.1 THE MOTION EQUATION OF AN OBJECT IN NEWTON'S SPACETIME

First let us discuss the motion equation of an object in Newton's spacetime. Let us assume that there exist only particle 1, particle 2,..., particle n and testing particle in whole space, a total of the $n + 1$ particles. Let $\mathbf{F}_1, \mathbf{F}_2 \dots \mathbf{F}_n$ be respectively the forces applied on the testing particle by the n particles. Let $m_1, m_2 \dots m_n$ be respectively the inertial masses of the testing particle by the n particles through the inertial interaction. Let $\mathbf{r}_1, \mathbf{r}_2 \dots \mathbf{r}_n$ be respectively the position vectors of the n particles relative to the testing particle. Let $\mathbf{V}_1 \equiv \frac{d\mathbf{r}_1}{dt}, \mathbf{V}_2 \dots \mathbf{V}_n \equiv \frac{d\mathbf{r}_n}{dt}$ be respectively the velocities of the n particles relative to the testing particle. We assume that the dynamical equations of the testing particle are the following three possibilities:

1)

$$\mathbf{F}_1 + \mathbf{F}_2 + \dots + \mathbf{F}_n = -\frac{d}{dt}(m_1\mathbf{V}_1 + m_2\mathbf{V}_2 + \dots + m_n\mathbf{V}_n). \quad (20)$$

There are n vectors $\mathbf{r}_1, \mathbf{r}_2 \dots \mathbf{r}_n$ to be determined. There are $n + 1$ equations in the form of equation (20) for the $n + 1$ particles, but the sum of the $n + 1$ equations is an identical equation $0 = 0$. Therefore, there are only the n independent equations

in the $n + 1$ equations. The n vector equations are complete or self-sufficiency for the n unknown vectors.

2)

$$\mathbf{F}_1 + \mathbf{F}_2 + \dots + \mathbf{F}_n = -\frac{d}{dt}(m_1 \mathbf{V}_1 + m_2 \mathbf{V}_2 + \dots + m_n \mathbf{V}_n + m_{in} \mathbf{U}). \quad (21)$$

where, m_{in} is the internal inertial interaction mass between each internal component of the testing particle, and m_{in} can be called as the internal inertial mass of the testing particle. \mathbf{U} is the velocity of the center of inertial masses of particle 1, particle 2, ..., particle n relative to the testing particle.

3)

$$\mathbf{F}_1 + \mathbf{F}_2 + \dots + \mathbf{F}_n = -\frac{d(m\mathbf{U})}{dt} = \frac{d(m\mathbf{V})}{dt}. \quad (22)$$

where, $m \equiv m_1 + m_2 + \dots + m_n + m_{in}$ is the total inertial mass of the testing particle. \mathbf{U} is the velocity of the center of inertial masses of particle 1, particle 2, ..., particle n relative to the testing particle. $\mathbf{V} \equiv -\mathbf{U}$ is the velocity of the testing particle relative to the center of inertial masses of particle 1, particle 2, ..., particle n .

Please note the following

[1] In equations (20) and (21), $\frac{d(m_i \mathbf{V}_i)}{dt}$ is the inertial force applied to the testing particle by the i -th particle. The testing particle also applies an inertial force $-\frac{d(m_i \mathbf{V}_i)}{dt}$ to the i -th particle. That is, inertial force satisfies Newton's third law.

[2] In the case of 1) and 2), we generally do not have the concept of the inertial reference frame. But when $\mathbf{V}_1 = \mathbf{V}_2 = \dots = \mathbf{V}_n \equiv -\mathbf{V}$, that is, no relative motions among the n particles, equations (20) and (21) become

$$\mathbf{F}_1 + \mathbf{F}_2 + \dots + \mathbf{F}_n = \frac{d}{dt}((m_1 + m_2 + \dots + m_n) \mathbf{V}) = \frac{d(m\mathbf{V})}{dt}. \quad (23)$$

$$\mathbf{F}_1 + \mathbf{F}_2 + \dots + \mathbf{F}_n = \frac{d}{dt}((m_1 + m_2 + \dots + m_n + m_{in}) \mathbf{V}) = \frac{d(m\mathbf{V})}{dt}. \quad (24)$$

Referring to the above, $m \equiv m_1 + m_2 + \dots + m_n$ is the total inertial mass of the testing particle as the result of the inertial interaction of the n particles. Or $m \equiv m_1 + m_2 + \dots + m_n + m_{in}$ is the total inertial mass of the testing particle. \mathbf{V} is the velocity of the testing particle relative to the n particles. Then, equations (23)

and (24) have the form of Newton's motion equation. In this case, the system of the n particles is the inertial reference frame. Equation (22) itself is a form of Newton's motion equation and, in this case, the system of the n particles is the inertial reference frame.

[3] In the case of 2) and 3), when m_{in} cannot be ignored, the star in spiral galaxy will deviate from the rotation curve of spiral galaxy.

[4] If there only exist a testing particle and particle 1 in whole space, equations (23), (24), and (22) become

$$\mathbf{F}_1 = \frac{d}{dt}(m_1 \mathbf{V}). \quad (25)$$

$$\mathbf{F}_1 = \frac{d}{dt}((m_1 + m_{in}) \mathbf{V}). \quad (26)$$

$$\mathbf{F}_1 = \frac{d}{dt}((m_1 + m_{in}) \mathbf{V}). \quad (27)$$

Equations (25), (26), and (27) are the form of Newton's motion equation. In this case, although particle 1 has a force $-\mathbf{F}_1$ applied by the testing particle, particle 1 is an inertial reference system.

[5] From equation (22), (23), or (24), we have

$$\mathbf{F} = \frac{d(m\mathbf{V})}{dt} = m \frac{d\mathbf{V}}{dt} + \mathbf{V} \frac{dm}{dt}. \quad (28)$$

If the re-definition of the force $\tilde{\mathbf{F}}$ is

$$\tilde{\mathbf{F}} := m \frac{d\mathbf{V}}{dt}. \quad (29)$$

then

$$\tilde{\mathbf{F}} = \mathbf{F} - \mathbf{V} \frac{dm}{dt}. \quad (30)$$

When an object is moving to the center of substance, that is $\frac{dm}{dt} > 0$, then $-\mathbf{V} \frac{dm}{dt}$ and \mathbf{V} are in opposite directions. On the contrary, when an object is away from the center of substance, that is $\frac{dm}{dt} < 0$, then $-\mathbf{V} \frac{dm}{dt}$ and \mathbf{V} is in the same direction. Therefore, the term $-\mathbf{V} \frac{dm}{dt}$ has a repulsion effect due to the inertial interaction; that is, the term $-\mathbf{V} \frac{dm}{dt}$ becomes a repulsive force.

Perhaps the accelerating expansion of the universe is related to this. The spaceship flying to or from the Milky Way will have an additional acceleration. The former will be discussed in §6 and the latter will be discussed in the following. For simplicity, let the Earth, the Sun, and the center of the Milky Way be still relative to each other, on the same straight line, and have the spaceship move along the line. From equation (30), we know that the additional acceleration of the spaceship due to the inertial interaction is

$$\begin{aligned}
\mathbf{a}_I &= -\frac{1}{m} \mathbf{V} \frac{dm}{dt} \\
&= -\frac{1}{m} \mathbf{V} \frac{d}{dt} [\alpha_U m_G + K \frac{M_M m_G}{r_M} + K \frac{M_\odot m_G}{r_\odot} + K \frac{M_E m_G}{r_E}] \\
&= \frac{1}{m} \mathbf{V} (K \frac{M_M m_G}{r_M} \frac{\dot{r}_M}{r_M} + K \frac{M_\odot m_G}{r_\odot} \frac{\dot{r}_\odot}{r_\odot} + K \frac{M_E m_G}{r_E} \frac{\dot{r}_E}{r_E})
\end{aligned} \tag{31}$$

here, m : the inertial mass of the spaceship; m_G : the gravitational mass of the spaceship; r_M , r_\odot and r_E : the distances of the spaceship from the centers of the Milky Way, the Sun, and the Earth. In the following three cases will be discussed:

(1) Near the Earth, due to $|\dot{r}_M| = |\dot{r}_\odot| = |\dot{r}_E|$, $\alpha_M \approx 10\alpha_\odot \approx 100\alpha_E$ and $\alpha_M \frac{1}{r_M} \approx 10^{-9}\alpha_\odot \frac{1}{r_\odot} \approx 10^{-12}\alpha_E \frac{1}{r_E}$, we can have

$$\mathbf{a}_I \approx \frac{1}{100} \alpha_M \frac{\dot{r}_E^2}{r_E} \mathbf{e}_r \tag{32}$$

where \mathbf{e}_r is the unit vector positioned from the center of the Milky Way to the spaceship. For example, assuming $V \equiv \dot{r}_E \sim \pm 10^3 \text{ms}^{-1}$ and $r_E \sim 10^7 m$, then we have $\mathbf{a}_I \sim 8.2 \times 10^{-5} \text{ms}^{-2} \mathbf{e}_r$.

(2) The spaceship is in the interior of solar system, but away from the Earth. Therefore, we have $\frac{M_\odot}{r_\odot^2} \gg \frac{M_E}{r_E^2}$ ($r_E > 10^{-2} r_\odot$ is enough). Then

$$\mathbf{a}_I \approx \frac{1}{10} \alpha_M \frac{\dot{r}_\odot^2}{r_\odot} \mathbf{e}_r \tag{33}$$

For example, assuming $V \equiv \dot{r}_\odot \sim \pm 10^4 \text{ms}^{-1}$ and $r_\odot \sim 10^{11} m$, then we have $\mathbf{a}_I \sim 8.2 \times 10^{-6} \text{ms}^{-2} \mathbf{e}_r$.

(3)The spaceship is moving into the Milky Way, but away from solar system. Therefore, we have $\frac{M_M}{r_M^2} \gg \frac{M_\odot}{r_\odot^2} \gg \frac{M_E}{r_E^2}$ ($r_\odot > 10^{-5}r_M$ is enough). Then

$$\mathbf{a}_I \approx \alpha_M \frac{\dot{r}_M^2}{r_M} \mathbf{e}_r \quad (34)$$

For example, assuming $V \equiv \dot{r}_M \sim \pm 10^5 m s^{-1}$ and $r_M \sim 10^{20} m$, then we have $\mathbf{a}_I \sim 8.2 \times 10^{-11} m s^{-2} \mathbf{e}_r$.

3.2 THE MOTION EQUATION AND THE ENERGY FORMULA OF AN OBJECT IN MINKOWSKI'S SPACETIME

In Minkowski's spacetime (M, η_{ab}) , let \tilde{m}_G be the gravitational mass of the testing particle and m be the inertial mass of the testing particle by the whole of substances in the spacetime other than itself through the inertial interaction. From the formula of inertial mass, m can be expressed as follows

$$m = f(P) \tilde{m}_G. \quad (35)$$

where, $P \in M$ is the spacetime point in which the testing particle is located, and $f(P)$ is the function of the space-time point that is determined by the inertial interaction of all the substances, excluding the testing particle itself, in the spacetime. $f(P)$ has nothing to do with \tilde{m}_G .

$m = \gamma m_0$ is seen by the special relativity. m_0 is the still inertial mass, $\gamma \equiv (1 - \frac{u^2}{c^2})^{-\frac{1}{2}}$, while u is the 3-velocity of the testing particle relative to the observer. On the Earth, we have $m = \tilde{m}_G$ and $m_0 = m_G$. m_G is the still gravitational mass of the testing particle, then we have $\tilde{m}_G = \gamma m_G$. It can be assumed that $\tilde{m}_G = \gamma m_G$ is generally applicable. Therefore,

$$m = f(P) \tilde{m}_G = f(P) \gamma m_G. \quad (36)$$

Perhaps, m_G depends on the intrinsic properties and the fundamental interactions including or except for inertial interactions of the particles which have no structure and compose the testing particle (i.e., elementary particles). While γ is decided by

the basic interactions outside of the testing particle in addition to the inertial interaction, $f(P)$ is determined by the inertial interaction outside the testing particle.

3.2.1 THE TESTING PARTICLE IS ELEMENTARY PARTICLE

4-force F^a of the testing particle can be defined as

$$F^a := U^b \partial_b (f(P) m_G U^a). \quad (37)$$

where, ∂_a is the derivative operator associated with Minkowski's metric η_{ab} . U^a is the 4-velocity of the testing particle. m_G is the still gravitational mass of the testing particle which has no structure. m_G is the intrinsic property of elementary particle. Of course, here we do not discuss quantum mechanics.

If there exists an inertial frame, meaning that all objects except the testing particle are relatively static, then we can assume that the motion equation of the testing particle for this coordinate system is

$$\mathbf{F} = \frac{d}{dt} (f(P) \gamma m_G \mathbf{u}). \quad (38)$$

where, \mathbf{u} and \mathbf{F} are respectively the 3-velocity and 3-force of the testing particle. t is the coordinate time of the inertial frame.

The following equations can be proved

$$F^i = \gamma f^i. \quad (39)$$

$$F^0 = \gamma \mathbf{f} \cdot \mathbf{u} + c^2 \gamma m_G \frac{d}{dt} f(P). \quad (40)$$

Please note the following: [1] When the relationships (39) and (40) between 4-force F^a and 3-force \mathbf{f} are known, equation (37) also can be called the motion equation of the testing particle. [2] If there is no any inertial frame, the relationship between 4-force and 3-force needs to be further studied; [3] For a free particle $\mathbf{f} = 0$, we have $F^i = 0$, yet $F^0 = c^2 \gamma m_G \frac{d}{dt} f(P) \neq 0$. From equation (37), we also know that 4-force $F^a \neq 0$ of the testing particle due to the inertial interaction.

From equation (38), we have that from t_1 to t_2 ,

$$\int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{u} dt + \int_{P_1}^{P_2} c\sqrt{c^2 - u^2} m_G df(P) = m_2 c^2 - m_1 c^2. \quad (41)$$

where, $m_1 = f(P_1)\gamma_1 m_G$ and $m_2 = f(P_2)\gamma_2 m_G$ are respectively the inertial masses of the testing particle at t_1 and t_2 . Like the special relativity theory, the energy of the elementary particles is defined as

$$E = mc^2 = f(P)\gamma m_G c^2. \quad (42)$$

If the Higgs mechanism really does exist, the mass which the particle obtains by the Higgs mechanism is likely to be the intrinsic gravitational mass m_G of the elementary particle.

3.2.2 TESTING PARTICLE HAVING INTERNAL STRUCTURE

For the testing particle which has internal structure, there are three possibilities:

1) Assume the motion equation of the testing particle be as follows (That is the definition formula of 4-force)

$$F^a := U^b \partial_b [(f(P)m_G + m_{in})U^a]. \quad (43)$$

where, m_G is the rest gravitational mass of testing particle, which is determined by the intrinsic gravitational mass of elementary particles composing the testing particle and the fundamental interactions except for inertial interactions of the elementary particles. $f(P)$ is decided by the inertial interaction outside of the testing particle. It is noticed that m_{in} is the internal inertial mass of the testing particle rather than the bound energy of the testing particle, and the bound energy of the testing particle is reflected in m_G .

It can be imagined that the testing particle is formed together by elementary particles from the dispersed state without inertial interaction. Then, it is reasonable to assume that the energy of the testing particle is

$$E = \gamma[f(P)m_G + m_{in}]c^2 = E_I + E_{in}. \quad (44)$$

here, $E_I \equiv \gamma f(P)m_G c^2$ and $E_{in} \equiv \gamma m_{in} c^2$, γ is decided by the basic interactions outside the testing particle except for the inertial interaction.

2) Assume the motion equation and the energy of the testing particle be respectively

$$F^a := U^b \partial_b [f(P)m_G U^a]. \quad (45)$$

$$E = \gamma [f(P)m_G + m_{in}] c^2 = E_I + E_{in}. \quad (46)$$

where, $E_I \equiv \gamma f(P)m_G c^2$ and $E_{in} \equiv \gamma m_{in} c^2$.

3) Assume the motion equation and the energy of the testing particle be respectively

$$F^a := U^b \partial_b [f(P)m_G U^a]. \quad (47)$$

$$E = E_I \equiv \gamma f(P)m_G c^2. \quad (48)$$

where, m_G is the gravitational mass of testing particle, which is determined by the intrinsic gravitational mass of elementary particles composing the testing particle and the fundamental interactions including inertial interactions of the elementary particles. The internal inertial interactions of the testing particle are reflected in the effects for m_G .

Please note the following:[1]The relationship between 4 force and 3 force needs to be further studied.[2] If there are only two particles in Minkowski's spacetime, then every particle is an inertial frame for the other particle. But the world lines are not geodesic due to the interaction force between the two. If the motion equation of the testing particle is the case 2) or 3), because the motion of the two particles is completely symmetrical, the proper time of the two particles are the same after the separation of the two particles and then to meet again. If the motion equation of the testing particle is case 1), because the motion of the two particles is not symmetrical, the proper time of the two particles are not the same after the separation of the two particles and then to meet again. Generally there does not exist inertial frame. The reference frame is an inertial frame only when there are no relative motions of all viewers in reference frame. In special relativity, describing the inertial system

by Lorenzian coordinate system is just an approximate case when the force of the inertial frame by the testing particle can be ignored. In case 1), this approximation, after all, is a good approximation. However, in case 2) and 3), ignoring the force of the inertial frame by the testing particle is not very plausible. This treatment can only be a practical way.

3.3 THE MOTION EQUATION OF AN OBJECT IN A GENERAL SPACETIME

Let (M, g_{ab}) be a general space-time

If the testing particle is an elementary particle, it is assumed that the motion equation (That is the definition formula of 4-force) and the energy of the testing particle be respectively

$$F^a := U^b \nabla_b (f(P) m_G U^a). \quad (49)$$

$$E = mc^2 = f(P) \gamma m_G c^2. \quad (50)$$

where, ∇_a is the derivative operator associated with the space-time's metric g_{ab} . F^a and U^a are respectively the 4-force and 4-velocity of the testing particle.

For the testing particle which have internal structure, there are also three possibilities corresponding to Minkowski's spacetime:

(1) Assume the motion equation (That is the definition formula of 4-force) and the energy of the testing particle be respectively

$$F^a := U^b \nabla_b [(f(P) m_G + m_{in}) U^a]. \quad (51)$$

$$E = \gamma [f(P) m_G + m_{in}] c^2 = E_I + E_{in}. \quad (52)$$

where, $E_I \equiv \gamma f(P) m_G c^2$ and $E_{in} \equiv \gamma m_{in} c^2$

(2) Assume the motion equation and the energy of the testing particle be respectively

$$F^a := U^b \nabla_b [f(P) m_G U^a]. \quad (53)$$

$$E = \gamma[f(P)m_G + m_{in}]c^2 = E_I + E_{in}. \quad (54)$$

where, $E_I \equiv \gamma f(P)m_G c^2$, and $E_{in} \equiv \gamma m_{in} c^2$.

(3) Assume the motion equation and the energy of the testing particle be respectively

$$F^a := U^b \nabla_b [f(P)m_G U^a]. \quad (55)$$

$$E_I = \gamma f(P)m_G c^2. \quad (56)$$

Please note:

1) Because the inertial interaction is different from the known four fundamental interactions, the inertial interaction directly impacts the energy of objects (this is reflected in $f(P)$ and m_{in} of the energy expression). The inertial interaction also produces the inertial force in equations (20) and (21) and the repulsive force in equation (30). Therefore perhaps the inertial interaction is a new interaction. Or at least it is another aspect of the gravitational interaction which people have not recognized yet.

2) From equation (49), (51), (53), and (55), we know that 4-force $F^a \neq 0$ of the testing particle due to the inertial interaction. The relationship between 4 force and 3 force needs to be further studied. Which one of the above three possibilities, or some other case, is the particle motion equation and energy should be determined by experiment and observation.

3) In the case (2) and (3), $\frac{\gamma f(P)m_G}{\gamma m_G} = f(P)$, the ratio of the inertial mass and the gravitational mass of an object is dependent on the point in space-time, instead of independent objects. In the case (1), $\frac{\gamma(f(P)m_G + m_{in})}{\gamma m_G} = f(P) + \frac{m_{in}}{m_G}$, the ratio of the inertial mass and the gravitational mass of an object is dependent on object. However, if $m_{in} \ll m_G$, $\frac{\gamma(f(P)m_G + m_{in})}{\gamma m_G} \approx f(P)$ is independent on object.

4) Assuming that $L(\tau)$ is the world line of any particle, τ is the proper time of the particle, U^a is the 4-velocity, and $P \in L$, then the 3-velocity of the particle relative to any instantaneous observer (P, Z^a) (Z^a is the 4-velocity the observer) can be defined as

$$u^a := h^a_b U^b / \gamma. \quad (57)$$

where, $h^a_b \equiv g^{ac}h_{cb}$, $h_{ab} \equiv g_{ab} + Z_a Z_b$, and $\gamma \equiv -U^a Z_a$.

The 3-speed of a particle relative to any instantaneous observer (P, Z^a) can be defined as

$$u := \sqrt{u^a u_a}. \quad (58)$$

where, $u_a := h_{ab}u^b$. It can be proved that (1) $\gamma = (1 - \frac{u^2}{c^2})^{-\frac{1}{2}}$; (2) $U^a = \gamma(Z^a + u^a)$.

5)The 4-momentum of a particle is defined as

$$P^a := f(P)m_G U^a. \quad (59)$$

The 4-momentum of a particle can be decomposed as 3+1 by an instantaneous observer (P, Z^a) .

$$P^a = E_I Z^a + p^a. \quad (60)$$

where, $p^a \equiv \gamma f(P)m_G u^a$ is the 3-momentum of a particle relative to an instantaneous observer (P, Z^a) , obviously, $E_I = -P^a Z_a$.

Or the 4-momentum of a particle is defined as

$$P^a := (f(P)m_G + m_{in})U^a. \quad (61)$$

The 4-momentum of a particle can be decomposed as 3+1 by an instantaneous observer (P, Z^a)

$$P^a = E Z^a + p^a. \quad (62)$$

where, $p^a \equiv \gamma(f(P)m_G + m_{in})u^a$ is the 3-momentum of a particle relative to an instantaneous observer (P, Z^a) , obviously, $E = -P^a Z_a$.

4 THEORY OF GRAVITY

4.1 EINSTEIN'S THEORY OF GRAVITY PERHAPS NEEDS TO BE MODIFIED

Einstein's theory of gravity perhaps needs to be modified. At least, there are three reasons. First, if the inertial interaction does exist, T_{ab} , the energy-momentum

tensor field of substances does not satisfy $\nabla^a T_{ab} = 0$; but Einstein tensor G_{ab} satisfies $\nabla^a G_{ab} = 0$. For example, the inertial mass of a given system, though without exchange of matter with the outside, will still be changed, because the outside matter changes distribution. It means that the continuity equation of inertial mass is not satisfied. Second, taking any cross section in the perfect fluid, the matters which lie on each side of the cross-section generate attractive effect due to the inertial mass density ρ and exclusion effect due to the pressure p . But in Einstein's equation, both the pressure $p > 0$ and the inertial mass density ρ generate attractive effect; only the pressure $p < 0$ generates exclusion effect. This can be found from Einstein's equation of the perfect fluid

$$R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi[(\rho + p)U_a U_b + pg_{ab}]. \quad (63)$$

The pressure p not only gives to the contribution of the second term, but also it appears in the first item, with the same sign as ρ , in the "source" of the right hand side in equation (63). Both p and ρ produce attractive effect. As a concrete example, the above can be seen from one of the Einstein's cosmic evolution equations[2], $3\ddot{a} = -4\pi a(\rho + 3p)$. Third, the Einstein's equation does not imply the inertial exclusion effect reflected in equation (30).

The inertial mass and the gravitational mass of an object are generally not equal due to the inertial interaction. It means that the equivalence principle about the equivalence of the inertial mass and the gravitational mass of an object perhaps does not hold. From equations (53), (55), and (51) we know that the ratio of inertial mass and gravitational mass, $\frac{\gamma f(P)m_G}{\gamma m_G} = f(P)$ or $\frac{\gamma(f(P)m_G + m_{in})}{\gamma m_G} \approx f(P)$, is spacetime point dependent, but non-object dependent or approximately non-object dependent. It also means that the ratio of inertial mass and gravitational mass of all objects, measured at the same point in spacetime, is the same. However, if measured at different points in spacetime, the ratio is different. "The ratio of inertial mass and gravitational mass of all objects, measured at the same point in spacetime, is the same" can be regarded as the generalized equivalence principle about the equivalence of the inertial mass and the gravitational mass.

Due to the generalized equivalence principle about the equivalence of the inertial mass and the gravitational mass, the world lines of free particles are non-object dependent. We also can assume that the world line of a free particle is geodesic. Therefore, the theory of gravity should be still the theory about geometry. But if the motion equation of a particle with internal structure is equation (51), the description to a particle with internal structure is approximate in the geometry theory of gravity .

4.2 TIDAL PHENOMENON

4.2.1 THE TIDAL PHENOMENON OF NEWTON'S THEORY OF GRAVITY IN NEWTON'S SPACETIME

In Newton's space-time, the relationship between the inertial mass m and the gravitational mass m_G is as follows.

$$m = f(\mathbf{r})m_G. \quad (64)$$

where, $f(\mathbf{r})$ is the function of a point in space which is determined by the inertial interaction of all the substances in the spacetime other than the testing particle itself. $f(\mathbf{r})$ has nothing to do with m_G . When the motion equation is equation (24) and if m_{in} can be ignored, then equation (64) applies approximately. In a small spatial extent, the variation of $f(\mathbf{r})$ with respect to \mathbf{r} can be ignored. It is an approximation that $f(\mathbf{r})$ does not change with \mathbf{r} .

Let $\mathbf{r}(t) \equiv x^i(t)\mathbf{e}_i$ and $\mathbf{r}(t) + \mathbf{w}(t) \equiv [x^i(t) + w^i(t)]\mathbf{e}_i$ be respectively the spatial position vector of adjacent particle 1 and 2 free-falling in the gravitational field. \mathbf{e}_i is the basis of Cartesian coordinate system. Then $\mathbf{w}(t) \equiv w^i(t)\mathbf{e}_i$ is the position vector of particle 2 relative to particle 1. $\frac{d^2\mathbf{w}}{dt^2}$ is the tidal acceleration of particle 2 relative to particle 1. Let ϕ be Newtonian gravitational potential. Then from the motion equations (23) or (24) and Newton's law of universal gravitation, we can obtain

$$f(\mathbf{r})\frac{d^2w^i}{dt^2} \approx -\frac{\partial^2\phi}{\partial x^j\partial x^i}|_{\mathbf{r}}w^j. \quad (65)$$

4.2.2 THE TIDAL PHENOMENON IN GENERAL SPACETIME

Let (M, g_{ab}) be a general spacetime. Let $\gamma_s(\tau)$ denote a smooth one-parameter family of the world lines of free particles in gravitational field and Σ denote the two-dimensional submanifold spanned by the curves $\gamma_s(\tau)$. (τ, s) can be the coordinates of Σ , and $[Z, w]^a = 0$, where, $Z^a \equiv (\frac{\partial}{\partial \tau})^a$ and $w^a \equiv (\frac{\partial}{\partial s})^a$. The relative velocity and the relative acceleration of infinitesimally nearby free particles can be respectively defined as

$$u^a = Z^b \nabla_b w^a. \quad (66)$$

and

$$a^a = Z^b \nabla_b u^a. \quad (67)$$

The following geodesic deviation equation can be proved:

$$a^c = -R_{abd}{}^c Z^a w^b Z^d. \quad (68)$$

4.3 THE EQUATION OF GRAVITATIONAL FIELD

Equation (65) can be rewritten as

$$a^c = (\frac{\partial}{\partial x^i})^c \frac{d^2 w^i}{dt^2} \approx -\frac{1}{f(\mathbf{r})} w^b \partial_b \partial^c \phi. \quad (69)$$

The comparison of equation (68) and (69) implies the following correspondence

$$R_{abd}{}^c Z^a Z^d \longleftrightarrow \frac{1}{f(\mathbf{r})} \partial_b \partial^c \phi. \quad (70)$$

Contracting the index c and b , we have

$$R_{ad} Z^a Z^d \longleftrightarrow \frac{1}{f(\mathbf{r})} \partial_b \partial^b \phi = \frac{1}{f(\mathbf{r})} \nabla^2 \phi = 4\pi \frac{1}{f(\mathbf{r})} \rho_G = \frac{1}{f^2(\mathbf{r})} 4\pi \rho. \quad (71)$$

where, ρ_G and $\rho = f(\mathbf{r})\rho_G$ are respectively the gravitational mass density and the inertial mass density of matter.

From the discussion in the beginning of §4.1, we have known that if there does exist inertial interaction then it is not the energy-momentum tensor T_{ab} to decide the geometry properties of spacetime.

Definition: the tensor field deciding the geometry properties of spacetime is called a "matter field tensor" , denoted as M_{ab} .

From equation (71) it can be assumed that M_{ab} satisfies

$$\rho = M_{ab}Z^aZ^b \quad (72)$$

ρ is the inertial mass density of matter measured by an observer Z^a . Then equation (71) becomes

$$R_{ad}Z^aZ^d \longleftrightarrow \frac{1}{f^2(\mathbf{r})}4\pi M_{ab}Z^aZ^b. \quad (73)$$

Please note the following

(1) Assume the matter field tensor of a perfect fluid be $M_{ab} = (\rho + f_1(p, \rho))U_aU_b + \tilde{f}_1(p, \rho)g_{ab}$, and we might as well take $f_1(p, \rho) = \tilde{f}_1(p, \rho)$, then

$$M_{ab} = (\rho + f_1(p, \rho))U_aU_b + f_1(p, \rho)g_{ab}. \quad (74)$$

where, $U^a = g^{ab}U_b$ is the 4-velocity field of perfect fluid. $\rho = M_{ab}U^aU^b$ is the inertial mass density of perfect fluid measured by a co-moving observer. p is the pressure of perfect fluid.

$$f_1(p, \rho) \equiv \hat{\beta}p + f_2(\rho). \quad (75)$$

$\hat{\beta}$ is a constant to be determined. Its value is discussed later. $f_2(\rho)$ is related to the change rate with proper time of the inertial mass and the speed of movement of perfect fluid's matter element, which will be discussed in the cosmology portion.

(2) The matter field tensor M_{ab} is different from the energy-momentum tensor T_{ab} , but both p and $f_2(\rho)$ can be ignored for the non-relativistic dust matter within the small scope of space. Then $M_{ab} \approx T_{ab}$.

(3) $\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V}$ in M_{ab} , ΔV is the volume element. Δm is the inertial mass in ΔV . Then ρ in M_{ab} contains m_{in} in equation (52) or (54).

(4) Due to the Lorentz Covariance of vacuum, i.e., the status of vacuum has nothing to do with the observer. It is reasonable to assume that vacuum has no inertial mass and gravitational mass. That is, assuming that the energy of vacuum

is not included in the energy of equations (52), (54), and (56), therefore, neither is it included in the matter field tensor M_{ab} . The energy of vacuum does not affect the property of spacetime. Furthermore, the Casimir force $F_{cas} \propto \frac{1}{d^4}$, d is the distance between the two flat conductors. When $d \rightarrow \infty$, then $F_{cas} \rightarrow 0$. It is reasonable to assume that the pressure of infinite vacuum is $p = 0$. Therefore, it is reasonable to assume that vacuum does not affect the property of spacetime.

(5) The need of negative pressure in the Einstein's theory may be due to the attractive effect of the pressure in the Einstein's theory. Because $p < 0$ appears only in the theoretical study of vacuum, cosmological constant, and the dark energy, the physical phenomenon of $p < 0$ is not directly observed and the physical image and physical essence of $p < 0$ is not clear. We assume $p \geq 0$ in this paper.

From equation (73), assume the equation of gravitational field be

$$R_{ab} + \alpha R g_{ab} + \Lambda g_{ab} = \tilde{\kappa} M_{ab}. \quad (76)$$

where, $\tilde{\kappa} \equiv \frac{\kappa}{f^2(P)}$, α , and κ are constants to be determined. Λ is the cosmological constant.

From equation (76), we have

$$R = \frac{\tilde{\kappa} M - 4\Lambda}{4\alpha + 1}. \quad (77)$$

where, $M \equiv M_a^a$. For a perfect fluid and any observer whose 4-velocity is Z^a , we have

$$R_{ab} Z^a Z^b = \frac{\gamma^2(4\alpha + 1) - \alpha}{4\alpha + 1} \tilde{\kappa} \rho + \tilde{\kappa} (\gamma^2 - \frac{\alpha + 1}{4\alpha + 1}) f_1(p, \rho) + \frac{\Lambda}{4\alpha + 1}. \quad (78)$$

where, $\gamma \equiv -U^a Z_a$. It is easy to prove that the energy-momentum tensor T_{ab} of the perfect fluid of $\rho \geq 0$ and $p \geq 0$ satisfies weak energy condition, strong energy condition and dominant energy condition. To avoid space-time singularity in our theory as much as possible, it is required that there is an observer Z^a so that in an evolutionary stage of the observer $R_{ab} Z^a Z^b < 0$ satisfies. It means that the conditions of singularity theorem are not satisfied.

For the dust matter within the small scope of space, when Λ can be ignored and Newtonian approximation applies, $\gamma \equiv -U^a Z_a \approx 1$ and $f_1(p, \rho) \approx 0$, we have

$$R_{ab} Z^a Z^b \approx \frac{3\alpha + 1}{4\alpha + 1} \tilde{\kappa} \rho. \quad (79)$$

Compared with the equation (71), it can be obtained

$$\kappa \frac{3\alpha + 1}{4\alpha + 1} = 4\pi. \quad (80)$$

From equation (80), we know $\alpha \neq -\frac{1}{3}$ and $\alpha \neq -\frac{1}{4}$.

4.4 THE EQUATION OF GRAVITATIONAL FIELD IN VACUUM

For vacuum $M_{ab} = 0$, ignoring Λ , the equation of gravitation (76) becomes

$$R_{ab} = 0. \quad (81)$$

It is the same as the Einstein's vacuum equation in form.

4.5 LINEARIZED THEORY OF GRAVITY AND THE NEWTONIAN LIMIT

4.5.1 LINEARIZED THEORY OF GRAVITY

In the case of a weak gravitational field, the metric g_{ab} of spacetime is very close to Minkowski metric η_{ab} . γ_{ab} can be defined as

$$g_{ab} = \eta_{ab} + \gamma_{ab}. \quad (82)$$

γ_{ab} is very small. Substituting equation (82) into the gravitational field equation (76), ignoring Λ and taking first order approximation, the linear gravitational field equation can be obtained

$$\partial^c \partial_{(a} \gamma_{b)c} - \frac{1}{2} \partial^c \partial_c \gamma_{ab} - \frac{1}{2} \partial_a \partial_b \gamma + \alpha \eta_{ab} (\partial^c \partial^d \gamma_{cd} - \partial^c \partial_c \gamma) = \tilde{\kappa} M_{ab}. \quad (83)$$

The equation (83) is the linearized equation of γ_{ab} and the linear approximation of the gravitational field equations.

There is a gauge freedom in any theory in which gravity is described by metric[2][3]. Let $\phi : M \rightarrow M$ be a diffeomorphism, then both (M, g_{ab}) and $(M, \phi^* g_{ab})$ represent the same physical spacetime. Thus, we can always find the vector field $\xi^a = \eta^{ab} \xi_b$. For the gauge transformation

$$\tilde{\gamma}_{cd} = \gamma_{cd} + \partial_a \xi_b + \partial_b \xi_a. \quad (84)$$

Let 00 component of equation (83)) in global inertial coordinate system[3] be divided into two equations

$$\partial^c \partial_c \gamma_{00} = -\frac{\sigma}{f^2(P)} M_{00}. \quad (85)$$

$$\partial^c \partial_{(0} \gamma_{0)c} - \frac{1}{2} \partial_0 \partial_0 \gamma + \alpha \eta_{00} (\partial^c \partial^d \gamma_{cd} - \partial^c \partial_c \gamma) = (\tilde{\kappa} - \frac{\sigma}{2f^2(P)}) M_{00}. \quad (86)$$

where σ is a constant to be determined. In fact, the 15 unknown functions $\gamma_{\mu\nu}$, U^μ and ρ (in Newton approximation, $p \approx 0$) can be obtained so that the 12 equations (83), (85), (86) and $g_{ab} U^a U^b = -1$ are satisfied.

4.5.2 THE NEWTONIAN LIMIT

From the following discussion we can see that Newton's theory of gravity can be regarded as a limiting case of the gravitational theory in the weak field under low-speed conditions.

(1) Weak field means equation (82).

(2) Low speed means that the field source and the objects move in low-speed.

The low-speed movement of the field source leads to the slow change of spacetime geometry, $\frac{\partial \gamma_{\mu\nu}}{\partial t} \approx 0$. The low-speed movement of objects also leads to that the 4-velocity of objects $U^a = (\frac{\partial}{\partial \tau})^a$ approximately equals to the 4-velocity of inertial observer $Z^a = (\frac{\partial}{\partial t})^a$ of the inertial coordinate system $\{t, x^i\}$. That is, the proper time τ of the object is approximately equal to the coordinate time t , $\tau \approx t$. From equation (72), we have

$$\rho = M_{ab} U^a U^b \approx M_{ab} Z^a Z^b = M_{00} \quad (87)$$

Substituting equation (87) into equation (85), we have

$$\nabla^2 \gamma_{00} = -\frac{\sigma}{f^2(P)} \rho = -\frac{\sigma}{f(P)} \rho_G. \quad (88)$$

As an approximation, ignoring the change of $f(P)$ with the point of space-time, equation (88) becomes

$$\nabla^2 \left(-\frac{4\pi f(P)}{\sigma} \gamma_{00} \right) \approx 4\pi \rho_G. \quad (89)$$

Comparing with Newtonian's gravity equation

$$\nabla^2 \phi = 4\pi \rho_G. \quad (90)$$

we have

$$\phi \approx -\frac{4\pi}{\sigma} f(P) \gamma_{00}. \quad (91)$$

The world line of a free particle in the gravitational field is geodesic. In inertial coordinate system $\{t, x^i\}$, the geodesic can be expressed as

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\sigma} \frac{dx^\nu}{d\tau} \frac{dx^\sigma}{d\tau} = 0. \quad (92)$$

where τ is the affine parameter of the geodesic. In the case of low-speed, $\frac{dx^i}{d\tau} = \frac{dx^i}{dt} \approx 0$, equation (92) becomes

$$\frac{d^2 x^\mu}{dt^2} = -\Gamma^\mu_{00}. \quad (93)$$

$$\Gamma^i_{00} = \frac{1}{2} \eta^{ij} (\gamma_{j0,0} + \gamma_{0j,0} - \gamma_{00,j}) \approx -\frac{1}{2} \frac{\partial \gamma_{00}}{\partial x^i} \approx \frac{\sigma}{8\pi f(P)} \frac{\partial \phi}{\partial x^i}. \quad (94)$$

Substituting equation (94) into equation (93), we have

$$\frac{d}{dt} [M_G f(P) \frac{dx^i}{dt}] \approx -\frac{\sigma}{8\pi} \frac{\partial (M_G \phi)}{\partial x^i}. \quad (95)$$

Comparing with Newtonian's gravity equation

$$\frac{d}{dt} [M_G f(P) \frac{dx^i}{dt}] = -\frac{\partial (M_G \phi)}{\partial x^i}. \quad (96)$$

we have

$$\sigma = 8\pi. \quad (97)$$

It is noticed that if let $\sigma = 8\pi$, Newton's theory of gravitation can be regarded as the weak gravitational field and the limit of low speed of our gravitation theory. Meanwhile, we have

$$\phi \approx -\frac{1}{2}f(P)\gamma_{00}. \quad (98)$$

5 STATIC SPHERICALLY SYMMETRIC METRIC

In static spherically symmetric spacetime (M, g_{ab}) , the line element of metric has the following form[2]

$$ds^2 = -e^{2A(r)}dt^2 + e^{2B(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (99)$$

where, $A(r)$ and $B(r)$ are the functions with respect to r to be determined.

5.1 SCHWARZSCHILD VACUUM SOLUTION

5.1.1 SCHWARZSCHILD VACUUM SOLUTION

Because the vacuum gravitational field equations (81) is the same as the Einstein's vacuum gravitational field equations, the static spherically symmetric vacuum metric still has the following form[2]

$$ds^2 = -(1 + \frac{C}{r})dt^2 + (1 + \frac{C}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (100)$$

Where C is a constant to be determined. By equation (98) and the Newtonian gravitational potential $\phi = -\frac{MG}{r}$ and ignoring the changes of $f(P)$ with point in spacetime, we can get $g_{00} = -1 - \frac{C}{r} = \eta_{00} + \gamma_{00} = -1 - \frac{2}{f(P)}\phi = -1 - (-\frac{2MG}{f(P)})\frac{1}{r}$ then $C = -\frac{2MG}{f(P)}$. Therefore, the static spherically symmetric vacuum metric is

$$ds^2 = -(1 - \frac{2MG}{r} \frac{1}{f(P)})dt^2 + (1 - \frac{2MG}{r} \frac{1}{f(P)})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (101)$$

Namely it is the line element of Schwarzschild spacetime. Please note that $f(P)$ is regarded as a constant in the range of spacetime discussed.

In the vacuum region of uniform spherical shell, we have $C = 0$ in equation (100). Otherwise, g_{00} and g^{11} at $r = 0$ will diverge. Then the metric in the vacuum region of uniform spherical shell is Minkowski metric.

5.1.2 THE EQUATIONS OF ELECTROMAGNETIC FIELD

For electromagnetic field, we assume

(1) Electromagnetic field is described by F_{ab} which satisfies

$$\nabla^a F_{ab} = -4\pi J_b. \quad (102)$$

$$\nabla_{[a} F_{bc]} = 0; \quad (103)$$

(2) From equation (103), we know that at least there exists 1-form \mathbf{A} locally, called electromagnetic 4-potential, so that $\mathbf{F} = d\mathbf{A}$. The motion equation of \mathbf{A} is

$$\nabla^a \nabla_a A_b - R_b^{d} A_d = -4\pi J_b; \quad (104)$$

(3) The energy-momentum tensor of electromagnetic field is

$$T_{ab} = \frac{1}{4\pi} (F_{ac} F_b^{c} - \frac{1}{4} g_{ab} F_{cd} F^{cd}); \quad (105)$$

(4) The matter field tensor of the electromagnetic field is to be studied.

(5) Let K^a be the 4-wavevector of photon. By any instantaneous observer (P, Z^a) at any point P of the spacetime, K^a at the point P may be decomposed as

$$K^a = \omega Z^a + k^a; \quad (106)$$

where

$$\omega = -K^a Z_a; \quad (107)$$

and k^a are respectively the angular frequency and 3-wavevector measured by the instantaneous observer (P, Z^a) .

(6) The 4-momentum P^a of photon is defined as

$$P^a := \hbar K^a; \quad (108)$$

(7) The world line of photon is a lightlike geodesic, and the affine parameter β satisfies

$$K^a = \left(\frac{\partial}{\partial \beta} \right)^a; \quad (109)$$

(8) By any instantaneous observer (P, Z^a) , P^a at any point P of the spacetime may be decomposed as

$$P^a = EZ^a + p^a. \quad (110)$$

where

$$E = \hbar\omega, \quad (111)$$

and

$$p^a = \hbar k^a; \quad (112)$$

They are respectively the energy and 3-momentum of the photon.

5.1.3 THE CONSERVED QUANTITY OF THE SCHWARZSCHILD SPACETIME

We can select Schwarzschild coordinates so that the parameter equations of world line $\gamma(\tau)$ of free particle are[2]

$$t = t(\tau), r = r(\tau), \theta = \frac{\pi}{2}, \varphi = \varphi(\tau). \quad (113)$$

Let $U^a = (\frac{\partial}{\partial\tau})^a$ be the tangent vector of a timelike geodesic $\gamma(\tau)$ and $U^a \equiv K^a = (\frac{\partial}{\partial\beta})^a$ be the tangent vector of a lightlike geodesic. Because $\xi^a \equiv (\frac{\partial}{\partial t})^a$ and $\xi_\varphi^a \equiv (\frac{\partial}{\partial\varphi})^a$ are the Killing vector fields of the Schwarzschild spacetime, we can define two constants on the timelike geodesic and the lightlike geodesic which are conserved quantities.

$$E := -g_{ab}(\frac{\partial}{\partial t})^a(\frac{\partial}{\partial\tau})^b = (1 - \frac{2M_G}{r} \frac{1}{f(P)}) \frac{dt}{d\tau}. \quad (114)$$

$$L := g_{ab}(\frac{\partial}{\partial\varphi})^a(\frac{\partial}{\partial\tau})^b = r^2 \frac{d\varphi}{d\tau}. \quad (115)$$

For photon, $\tau \equiv \beta$ in the above equation. $\xi^a \equiv (\frac{\partial}{\partial t})^a$ is the static Killing vector field of Schwarzschild spacetime. The 4-velocity of a static observer is

$$Z^a = \chi^{-1} \xi^a. \quad (116)$$

where, $\chi \equiv (-\xi^b \xi_b)^{1/2}$. from

$$E = -\xi_a U^a = \frac{\chi}{f(P)m_G} E_I. \quad (117)$$

We know that the conserved quantity E of a free particle is the energy E_I per unit rest inertial mass measured by a static observer in infinity. For free photons, from equations (108) and (110) we can obtain

$$E := -g_{ab}\xi^a K^b = -g_{ab}\xi^a P^b/\hbar = \chi E_{ph}/\hbar. \quad (118)$$

Where, $E_{ph} = \hbar\omega$ is the energy of photon measured by a static observer Z^a . From equation (118), we know that $\hbar E$ of free photons is the photon energy measured by infinity static observer.

Let $P \in \gamma(\tau)$, Z^a is the static observer at point P . Normalizing the coordinate basis at the point P gives the orthonormal tetrad in tangent space V_P at the point P [2].

$$\begin{aligned} (e_0)^a &\equiv (1 - \frac{2MG}{r} \frac{1}{f(P)})^{-1/2} (\frac{\partial}{\partial t})^a, (e_1)^a \equiv (1 - \frac{2MG}{r} \frac{1}{f(P)})^{1/2} (\frac{\partial}{\partial r})^a \\ (e_2)^a &\equiv r^{-1} (\frac{\partial}{\partial \theta})^a, (e_3)^a \equiv r^{-1} (\frac{\partial}{\partial \varphi})^a. \end{aligned} \quad (119)$$

Its dual 4-frame is

$$\begin{aligned} (e^0)_a &\equiv (1 - \frac{2MG}{r} \frac{1}{f(P)})^{1/2} (dt)_a, (e^1)_a \equiv (1 - \frac{2MG}{r} \frac{1}{f(P)})^{-1/2} (dr)_a \\ (e^2)_a &\equiv r (d\theta)_a, (e^3)_a \equiv r (d\varphi)_a. \end{aligned} \quad (120)$$

The angular momentum of free particle $\gamma(\tau)$ with respect to the static observer is defined (taking equation (53) or (51) and ignoring m_{in}) as

$$j^a := \varepsilon^a_{bc} \gamma f(P) m_G r^b u^c. \quad (121)$$

where, $r^b \equiv r(e_1)^b$. Then we have

$$j_a = \varepsilon_{abc} \gamma f(P) m_G r^b u^c = -\gamma f(P) m_G r u^3 (e^2)_a. \quad (122)$$

The magnitude of the angular momentum is

$$j := |\gamma f(P) m_G r u^3| = f(P) m_G |r^2 \frac{d\varphi}{d\tau}| = f(P) m_G |L|. \quad (123)$$

That is, $|L|$ is the magnitude of the 3-angular momentum with respect to the static reference frame per unit rest inertial mass of a free particle. The change of $f(P)$ with

P can be ignored in a small space range. If L is conserved, j is approximately also conserved. $\hbar L$ is the angular momentum of a photon in lightlike geodesic[3].

Let $U^a = (\frac{\partial}{\partial \tau})^a$ or $U^a \equiv K^a = (\frac{\partial}{\partial \beta})^a$ is the tangent vector on the world line $\gamma(\tau)$ of a free particle or the lightlike geodesic. Define

$$-\kappa := g_{ab}U^aU^b = -(1 - \frac{2M_G}{r}\frac{1}{f(P)})(\frac{dt}{d\tau})^2 + (1 - \frac{2M_G}{r}\frac{1}{f(P)})^{-1}(\frac{dr}{d\tau})^2 + r^2(\frac{d\varphi}{d\tau})^2. \quad (124)$$

Substituting equations (114) and (115) into equation (124), we have

$$-\kappa = -(1 - \frac{2M_G}{r}\frac{1}{f(P)})^{-1}E^2 + (1 - \frac{2M_G}{r}\frac{1}{f(P)})^{-1}(\frac{dr}{d\tau})^2 + \frac{L^2}{r^2}. \quad (125)$$

In above equation, for a photon $\equiv \tau \equiv \beta$ and $\kappa = \begin{cases} 1, & \text{the world line of a free particle} \\ 0, & \text{the lightlike geodesic} \end{cases}$

5.1.4 GRAVITATIONAL REDSHIFT

Let G and G' be two observers of arbitrary stationary reference frame in arbitrary stationary spacetime, and the photon emitted at the moment P by G reaches G' [2] at the moment P' . Let Z^a represent the 4-velocity of the observer, and K^a represent the 4-wavevector of the photon. Then from equation (107), we know that the angular frequencies of the photon with respect to the stationary observers at P and P' are respectively

$$\omega = -(K^a Z_a)|_P, \omega' = -(K^a Z_a)'|_{P'}. \quad (126)$$

The world line of the stationary observer coincides with the integral curves of the Killing vector field ξ^a , namely equation (116) applicable. As $K_a \xi^a$ is a constant in the lightlike geodesic, so

$$\frac{\omega'}{\omega} = \frac{\chi}{\chi'}. \quad (127)$$

For Schwarzschild spacetime

$$\chi^2 = -\xi^b \xi_b = -g_{00} = 1 - \frac{2M_G}{r}\frac{1}{f(P)}. \quad (128)$$

then

$$\frac{\lambda'}{\lambda} = (1 - \frac{2M_G}{r'}\frac{1}{f(P')})^{1/2} (1 - \frac{2M_G}{r}\frac{1}{f(P)})^{-1/2}. \quad (129)$$

Here, $f(P)$ and $f(P')$ are considered to be equal.

5.1.5 PERIHELION PRECESSION

In Newton's theory, from equation (28) we have

$$\mathbf{F} \cdot d\mathbf{r} = m \frac{d\mathbf{V}}{dt} \cdot d\mathbf{r} + \mathbf{V} \cdot d\mathbf{r} \frac{dm}{dt} = d\left(\frac{1}{2}mv^2\right) + v^2 dm. \quad (130)$$

That is, the kinetic energy theorem of Newton's theory is not established. The mechanical energy of the system, without external forces and non-conservative internal forces, is not conserved.

$$\mathbf{r} \times \mathbf{F} = \mathbf{r} \times \frac{d(m\mathbf{V})}{dt} = \frac{d(\mathbf{r} \times m\mathbf{V})}{dt} = \frac{d\mathbf{p}_L}{dt}. \quad (131)$$

in which, $\mathbf{p}_L \equiv \mathbf{r} \times m\mathbf{V}$ is the angular momentum of a particle with respect to the origin. Then the theorem of angular momentum and the law of angular momentum conservation are still valid.

The angular momentum $\mathbf{p}_L = \mathbf{r} \times m\mathbf{V} = f(\mathbf{r})m_G\mathbf{r} \times \mathbf{V}$ of the Mercury is conserved in the solar gravitational field. Then the $p_L = f(\mathbf{r})m_Gr^2\frac{d\varphi}{dt}$ is constant. Thus

$$L \equiv r^2 \frac{d\varphi}{dt} = p_L / [f(\mathbf{r})m_G]. \quad (132)$$

is approximately a constant. When $\nabla f(\mathbf{r})$ is ignored, the mechanical energy of the Mercury is approximately conserved. Namely

$$A = \frac{1}{2}m(u_r^2 + u_\varphi^2) + \left(-\frac{M_G m_G}{r}\right). \quad (133)$$

A is a constant. By equations (132) and (133) we can obtain

$$\frac{d^2\mu}{d\varphi^2} + \mu = M_G/[f(\mathbf{r})L^2]. \quad (134)$$

in which, $\mu \equiv 1/r$. The solution of equation (134) is

$$\mu(\varphi) = \frac{M_G}{f(\mathbf{r})L^2}(1 + e\cos\varphi). \quad (135)$$

Where, e is the integral constant and is the eccentricity.

Considering the case of the curved spacetime, ignoring the variation of $f(P)$ with respect to P , and taking $\kappa = 1$ in equation (125), from equations (125) and (115) we can obtain

$$\frac{d^2\mu}{d\varphi^2} + \mu = M_G/[f(P)L^2] + \frac{3M_G}{f(P)}\mu^2. \quad (136)$$

where $\mu \equiv 1/r$. Equation (135) can be used as the 0-order approximate solution of equation (136), i.e.

$$\mu_0(\varphi) = \frac{M_G}{f(\mathbf{r})L^2}(1 + e\cos\varphi). \quad (137)$$

The 1-order approximation of equation (136) is

$$\frac{d^2\mu_1}{d\varphi^2} + \mu_1 = M_G/[f(P)L^2] + \frac{3M_G}{f(P)}\mu_0^2 = M_G/[f(P)L^2] + \frac{3M_G^3}{f^3(P)L^4}(1 + 2e\cos\varphi + e^2\cos^2\varphi). \quad (138)$$

Then, by using the same solving method[2] with Einstein's case perihelion precession angle in a period can be obtained

$$\Delta\varphi_P \approx 6\pi M_G^2/[f^2(P)L^2]. \quad (139)$$

5.1.6 THE BENDING OF LIGHT

Let $\kappa = 0$ in equation (125). From equations (125) and (115) we have

$$\left(\frac{dr}{d\varphi}\right)^2 - \frac{E^2 r^4}{L^2} + r^2\left(1 - \frac{2M_G}{r} \frac{1}{f(P)}\right) = 0. \quad (140)$$

From equation (140) we have

$$\frac{d^2\mu}{d\varphi^2} + \mu = \frac{3M_G}{f(P)}\mu^2. \quad (141)$$

By using the same solving method[2] as the Einstein's case, the deflection angle of distant photon through a vacuum spherically symmetric gravitational field can be obtained

$$\beta \approx \frac{4M_G}{lf(P)}. \quad (142)$$

in which, l is an integral constant and it is approximately the distance from the center of spherically symmetric gravity to the track of the distant photon when photon track is not deflected.

The external regions of quasars, galaxies, and clusters of galaxies are nearly the cosmic background, $f(P) \approx 0.09 \sim 0.17$. Therefore, the effects of gravitational lens of quasars, galaxies, and clusters of galaxies are the $5.9 \sim 11$ times as big as those of the Einstein's theory.

In addition, if quasar is an Active Galactic Nuclei, its mass should not be less than the mass of the galaxy, and its effect of gravitational lens should not be weaker than that of the galaxy, rather than only be $5.9 \sim 11$ times higher than the effects of gravitational lens of ordinary stars.

5.2 THE INTERIOR STATE EQUATION OF STATIC SPHERICALLY SYMMETRIC STAR

When the stellar interior matter field is regarded as a perfect fluid, the matter field tensor is as equation (74). Ignoring Λ and substituting equations (99) and (74) into the gravitational field equation (76), we have the following equations which static spherically symmetric stellar internal metric satisfies

$$e^{-2B}[-(1+2\alpha)A''+(1+2\alpha)A'B'-(1+2\alpha)A'^2-(1+2\alpha)2r^{-1}A'+4\alpha r^{-1}B'-2\alpha r^{-2}]+2\alpha r^{-2} = -\tilde{\kappa}\rho. \quad (143)$$

$$e^{-2B}[-(1+2\alpha)A''+(1+2\alpha)A'B'-(1+2\alpha)A'^2+(1+2\alpha)2r^{-1}B'-4\alpha r^{-1}A'-2\alpha r^{-2}]+2\alpha r^{-2} = \tilde{\kappa}f_1(p, \rho). \quad (144)$$

$$-e^{-2B}[(1+2\alpha)r^{-2}+(1+4\alpha)r^{-1}(A'-B')+2\alpha A''-2\alpha A'B'+2\alpha A'^2]+(1+2\alpha)r^{-2} = \tilde{\kappa}f_1(p, \rho). \quad (145)$$

Where, $A' \equiv \frac{dA}{dr}$, $A'' \equiv \frac{d^2A}{dr^2}$, and $B' \equiv \frac{dB}{dr}$. When $\alpha = -\frac{1}{2}$, $\tilde{\kappa} = 8\pi$, and $f_1(p, \rho) = p$, the equations (143), (144), and (145) return to the case of Einstein[2]. Static spherically symmetric stellar internal state is decided by the four functions $A(r)$, $B(r)$, $\rho(r)$, and $p(r)$, which satisfy the three equations (143), (144), (145), and the state equation $F(p, \rho) = 0$.

Let us discuss the following

(1) From equations (143), (144), and (145), we can obtain

$$2r^{-1}(B' + A')e^{-2B} = \tilde{\kappa}[f_1(p, \rho) + \rho]. \quad (146)$$

$$e^{-2B}[-(1+2\alpha)r^{-1}B' + (1+6\alpha)r^{-1}A' + (1+4\alpha)r^{-2}] - (1+4\alpha)r^{-2} = -\tilde{\kappa}f_1(p, \rho). \quad (147)$$

From equations (146) and (147), we have

$$\frac{d}{dr}(re^{-2B}) = 1 - \frac{3(1+2\alpha)}{2(1+4\alpha)}\tilde{\kappa}r^2f_1(p, \rho) - \frac{1+6\alpha}{2(1+4\alpha)}\tilde{\kappa}\rho r^2. \quad (148)$$

In the non-relativistic case, $p \ll \rho$. In the static case, $f_2(\rho) = 0$. By integrating equation (148), we can obtain

$$e^{-2B} \approx 1 - \frac{1+6\alpha}{1+4\alpha} \frac{\tilde{\kappa}}{8\pi} \frac{m(r)}{r}. \quad (149)$$

where

$$m(r) \equiv 4\pi f^2(\mathbf{r}) \int_0^r \frac{\rho_G(r)}{f(\mathbf{r})} r^2 dr. \quad (150)$$

$\rho_G(r)$ is the gravitational mass density of the star.

$$g_{11}(r) = e^{2B} \approx [1 - \frac{1+6\alpha}{1+4\alpha} \frac{\tilde{\kappa}}{8\pi} \frac{m(r)}{r}]^{-1}. \quad (151)$$

(2) $\nabla^a T_{ab} = 0$ is approximately established in the case of Newton approximation.

Thus, we obtain

$$\frac{dp}{dr} \approx -(p + \rho) \frac{dA}{dr}. \quad (152)$$

In the non-relativistic case, $p \ll \rho$. In the static case, $f_2(\rho) = 0$. From equations (146), (151), and (152), we can obtain

$$\frac{dp}{dr} \approx \rho B' - \frac{1}{2}\tilde{\kappa}\rho^2 r e^{-2B}. \quad (153)$$

Considering that the inertial mass density of the star is nearly uniform, namely, $\rho \sim \bar{\rho}$, and the change of $f(\mathbf{r})$ with respect to \mathbf{r} can be ignored, then we have $m(r) \approx \frac{4\pi r^3}{3}\rho$. From equation (149) we obtain

$$B' \approx \frac{1+6\alpha}{1+4\alpha} \frac{\tilde{\kappa}}{16\pi} (4\pi\rho r - \frac{m}{r^2}) e^{2B}. \quad (154)$$

From equations (149), (153), (154), and taking (80) into account, we can obtain

$$\frac{dp}{dr} \approx -\frac{m_G \rho_G}{r^2}. \quad (155)$$

That is the results of Newtonian mechanics. The pressure $p > 0$ is the exclusion effect in the Newtonian's theory, but the positive pressure of $p > 0$ produces gravitational effects in Einstein's gravitational field equation. Therefore, it may be unreasonable that the Newton's form like (155) appears in the Einstein's theory.

(3) From equation (147), we have

$$A' = \frac{1+2\alpha}{1+6\alpha}B' - \frac{1}{1+6\alpha}\tilde{\kappa}r f_1(p, \rho)e^{2B} + \frac{1+4\alpha}{1+6\alpha}r^{-1}(e^{2B} - 1). \quad (156)$$

Ignoring the item $f_1(p, \rho)$ in equation (156) and taking up to 1-order approximation of $\frac{m(r)}{r}$, we can obtain

$$A' \approx \frac{1}{3} \frac{1+3\alpha}{1+4\alpha} \tilde{\kappa} \rho r. \quad (157)$$

Considering that the inertial mass density of the star is nearly uniform and integrating equation (157), we can obtain

$$g_{00} = -e^{2A} \approx -C e^{\frac{1+3\alpha}{1+4\alpha} \frac{\tilde{\kappa}}{4\pi} \frac{m(r)}{r}}. \quad (158)$$

C is an integral constant. In internal vacuum region of uniform shell, $\rho = 0$, from equations (151) and (158) we know that the metric of the region is Minkowski metric.

(4) From the gravitational field equation of $\Lambda = 0$

$$R_{ab} + \alpha R g_{ab} = \tilde{\kappa} M_{ab}. \quad (159)$$

and $\nabla^a G_{ab} = 0$, considering the approximation of $\nabla^a \tilde{\kappa} \approx 0$, we have

$$\left(\frac{\partial}{\partial r}\right)^b \nabla^a M_{ab} \approx \frac{1+2\alpha}{2\tilde{\kappa}} \frac{dR}{dr}. \quad (160)$$

From equation (74) we can obtain

$$\left(\frac{\partial}{\partial r}\right)^b \nabla^a M_{ab} = (\rho + f_1(p, \rho)) \frac{dA}{dr} + \frac{df_1(p, \rho)}{dr}. \quad (161)$$

From equations (75), (80), (143), (144), (156), (157), (160), (161), and considering Newton approximation and $\rho \sim \bar{\rho}$, i.e. $\frac{d}{dr} \frac{m}{r^3} \sim \frac{d\rho}{dr} \approx 0$, we have

$$\frac{6\alpha - 1}{2(1+6\alpha)} \hat{\beta} \frac{dp}{dr} \approx -\frac{m_G \rho_G}{r^2}. \quad (162)$$

Comparing with equation (155) we can obtain

$$\hat{\beta} = \frac{2(1+6\alpha)}{6\alpha-1}. \quad (163)$$

(5) To avoid space-time singularity as much as possible in our theory, it is required that there be the observer whose 4-velocity is Z^a so that $R_{ab}Z^aZ^b < 0$ is satisfied, thereby the condition of singularity theorem does not satisfy. In equation (78), $\gamma \geq 1$. if $R_{ab}Z^aZ^b < 0$ is established, γ should be as small as possible. Let $Z^a = U^a$, i.e. $\gamma = 1$. Ignoring item Λ , then from equation (78), we know that the establishment of $R_{ab}Z^aZ^b < 0$ means

$$\frac{3\alpha+1}{4\alpha+1}\tilde{\kappa}\rho + \tilde{\kappa}\frac{3\alpha}{4\alpha+1}f_1(p, \rho) < 0. \quad (164)$$

It can be seen from equation (167) later in cosmology that equation (164) means the accelerating expansion of the universe. The first mass item of equation (164) is the attraction effect, then $\frac{3\alpha+1}{4\alpha+1}\tilde{\kappa} > 0$. The second pressure and inertia item is the exclusion effect, then $\tilde{\kappa}\frac{3\alpha}{4\alpha+1}\hat{\beta} < 0$. Under the condition $\kappa < 0$, we have $\frac{3\alpha+1}{4\alpha+1} < 0$ and $-\frac{1}{3} < \alpha < -\frac{1}{4}$. From the cosmology section we know that to get the accelerating expansion evolution equation of the universe, under the condition $\alpha > -\frac{1}{3}$, there must be $-\frac{1}{4} < \alpha < 0$. As such, there are only $\kappa > 0$ and $\frac{3\alpha+1}{4\alpha+1} > 0$. Further more there are $\alpha < -\frac{1}{3}$ or $\alpha > -\frac{1}{4}$. While $\alpha < -\frac{1}{3}$, $\tilde{\kappa}\frac{3\alpha}{4\alpha+1}\hat{\beta} = \tilde{\kappa}\frac{3\alpha}{4\alpha+1}\frac{2(1+6\alpha)}{6\alpha-1} > 0$, then it is $-\frac{1}{4} < \alpha < 0$. From $\frac{3\alpha}{4\alpha+1}\hat{\beta} = \frac{3\alpha}{4\alpha+1}\frac{2(1+6\alpha)}{6\alpha-1} < 0$ we have $\alpha < -\frac{1}{6}$. Therefore, there must be $-\frac{1}{4} < \alpha < -\frac{1}{6}$.

(6) Due to $\frac{1+6\alpha}{1+4\alpha} < 0$ in equation (151), from equations (151) and (158) we know that there are no horizon and one-way membrane region (i.e. the region of collapse) in the interior of spherically symmetric star. In the evolution process of spherically symmetric star, even if the matter of the star contracts and gets into the event horizon of Schwarzschild vacuum solution, the collapse singularity not necessarily will form. But due to the energy loss of the Hawking radiation, the matter will continue to shrink so that the divergent singularity may be formed. If some aspects of quantum field theory are not correct so that there is no the Hawking radiation, then the matter of the star will eventually stop the contraction. In this case although the

spherically symmetric star has evolved as a black hole, there is no singularity within the black hole. The reason of no singularity in the black hole is that in some stage of the evolution in spherically symmetric star, the second item of equation (164) leads to that $R_{ab}U^aU^b < 0$ is satisfied for the co-moving observer U^a with the matter of the star and the conditions of singularity theorem do not satisfy. But please pay attention to that on the surface of star, metrics inside and outside the star are not continuous. It seems that we can choose α so that $[1 - \frac{1+6\alpha}{1+4\alpha} \frac{\tilde{\kappa}}{8\pi} \frac{m(r)}{r}]^{-1}$ of equation (151) is equal to the $[1 - 2\frac{M_G}{rf(r)}]^{-1}$ of Schwarzschild vacuum solutions at $r = R$. Namely, $\frac{1+6\alpha}{1+4\alpha} \frac{\tilde{\kappa}}{8\pi} \frac{m(R)}{R} = \frac{2M_G}{Rf(R)}$. Wherein $M_G \equiv 4\pi \int_0^R \rho_G(r)[1 - \frac{1+6\alpha}{1+4\alpha} \frac{\tilde{\kappa}}{8\pi} \frac{m(r)}{r}]^{-\frac{1}{2}} r^2 dr$ is the gravitational mass of the star. But α , thus obtained, is dependent on $\rho_G(r)$ and $f(r)$. Therefore, it is inevitable that, on the star surface, the metrics inside and outside are not continuous. If vacuum is regarded as a special case of $\rho_G(r) = 0$, and the upper limit r of integrations of $m(r)$ in equations (151) and (158) enters the vacuum region, then the metric thus obtained will be unlike the vacuum metric. This shows that matter region and the vacuum region should be described by using the corresponding field equations respectively and the field equations can not generally cross the interface of the matter and the vacuum. The metrics on the interface of matter and vacuum are not necessarily discontinuous. For example, the metrics on the interface of the vacuum inside a uniform matter spherical shell and the spherical shell matter are continuous. It is associated with the pressure from the physical point of view that the metrics on the interface of matter and vacuum may not be continuous.

6 THE EVOLUTION OF THE UNIVERSE

6.1 THE DYNAMIC EQUATION OF THE UNIVERSE

6.1.1 THE ESSENTIAL EQUATIONS OF EVOLUTION OF $a(t)$

Let us assume that the cosmological principle is applicable, then the metric of the universe is the Robertson-Walker metric (2)[2]. The contents of universe can be

regarded as a perfect fluid. Substituting Robertson-Walker metric (2) and the matter field tensor (74) of perfect fluid into the gravitational field equation (76), we can obtain

$$-\frac{3\ddot{a}}{a} = \tilde{\kappa} \frac{(1+3\alpha)\rho + 3\alpha f_1(p, \rho)}{1+4\alpha} + \frac{\Lambda}{1+4\alpha}. \quad (165)$$

$$a\ddot{a} + 2\dot{a}^2 + 2k = \tilde{\kappa} \frac{\alpha\rho + (1+\alpha)f_1(p, \rho)}{1+4\alpha} a^2 - \frac{\Lambda}{1+4\alpha} a^2. \quad (166)$$

From equation (165) we know that if let $\alpha > -\frac{1}{4}$, cosmological constant Λ has an attraction rather than repulsion effect. Considering the case of cosmological constant $\Lambda = 0$, the equations (165) and (166) become

$$-\frac{3\ddot{a}}{a} = \tilde{\kappa} \frac{(1+3\alpha)\rho + 3\alpha f_1(p, \rho)}{1+4\alpha}. \quad (167)$$

$$a\ddot{a} + 2\dot{a}^2 + 2k = \tilde{\kappa} \frac{\alpha\rho + (1+\alpha)f_1(p, \rho)}{1+4\alpha} a^2. \quad (168)$$

Equations (167) and (168) are the essential equations to determine the evolution of the scale factor $a(t)$ of the universe. When $\alpha = -\frac{1}{2}$, $\tilde{\kappa} = 8\pi$, and $f_1(p, \rho) = p$, the equations (167) and (168) return to the Einstein's theory[2].

6.1.2 THE CRITICAL DENSITY OF INERTIAL MASS OF THE UNIVERSE

Discussing three cases below

$$(1) \ddot{a} > 0$$

Due to $-\frac{1}{4} < \alpha < 0$, and from equation (167), we have

$$(1+3\alpha)\rho + 3\alpha f_1(p, \rho) < 0. \quad (169)$$

From equations (167), (168), and (169), we can obtain

$$\rho < \rho_c + \frac{-6\alpha k}{\tilde{\kappa} G a^2}. \quad (170)$$

Where, $\rho_c \equiv \frac{-16\pi\alpha f^2(P)}{\kappa} \rho_{cE}$ is the critical density of cosmic inertial mass. $\rho_{cE} \equiv \frac{3H^2}{8\pi G}$ is the critical density of the universe mass in Einstein's theory.

$$(2)\ddot{a} = 0$$

$$\rho = \rho_c + \frac{-6\alpha k}{\tilde{\kappa}Ga^2}. \quad (171)$$

$$(3)\ddot{a} < 0$$

$$\rho > \rho_c + \frac{-6\alpha k}{\tilde{\kappa}Ga^2}. \quad (172)$$

Let us take $k = 0$, namely consider the flat universe. $\ddot{a} > 0$ results in $\rho < \rho_c$; $\ddot{a} = 0$ results in $\rho = \rho_c$; $\ddot{a} < 0$ results in $\rho > \rho_c$. That is, when the universe is flat, ρ takes its value in an interval containing ρ_c ; there is no the flatness problem in Einstein's theory. If we take $\alpha = -\frac{1}{5}$, today $f(P) = \alpha_U = 0.09 \sim 0.1$, then the critical density of the universe inertial mass today is $\rho_c \sim 0.018\rho_{cE}$ which is approximately 1.8% of the value in Einstein's theory. Today the universe is accelerating expansion, hence, the inertial mass density of the universe is $\rho < 1.8\%\rho_{cE}$.

6.2 THE EVOLUTION OF $a(t)$

Substituting

$$f^2(P) = f(P)\frac{4\pi K\rho_G}{\delta^2} = \frac{4\pi K}{\delta^2}\rho. \quad (173)$$

and $\tilde{\kappa} = \kappa/\frac{4\pi K}{\delta^2}\rho$ into equations (167) and (168), we have

$$\frac{1+2\alpha}{\alpha}\frac{1+3\alpha}{1+4\alpha}\frac{\ddot{a}}{\beta a} + \frac{2(1+3\alpha)}{1+4\alpha}\frac{1}{\beta}\left(\frac{\dot{a}}{a}\right)^2 + \frac{2(1+3\alpha)}{1+4\alpha}\frac{k}{\beta a^2} + \frac{1}{3\alpha} = 0. \quad (174)$$

where $\beta \equiv \frac{\delta^2}{K}$. Let us take into account the case of $k = 0$. Due to $\alpha > -\frac{1}{3}$ and solving equation (174), we can obtain

$$a(t) = A|\cos[\omega(t-t_0) + \varphi]|^n. \quad (175)$$

Where, $A = a_0/|\cos[\arctan\sqrt{\frac{3(1+3\alpha)}{\beta}}y_0]|^n$, t_0 is the value of t at a certain moment, $y_0 \equiv \frac{\dot{a}}{a}|_{t=t_0}$, $a_0 \equiv a(t_0)$, $\omega \equiv \frac{1+4\alpha}{1+2\alpha}\sqrt{\frac{\beta}{3(1+3\alpha)}}$, $\varphi \equiv -\arctan\sqrt{\frac{3(1+3\alpha)}{\beta}}y_0$, and $n \equiv \frac{1+2\alpha}{1+4\alpha}$.

Without loss of generality, we consider $-\frac{\pi}{2} \leq \omega(t-t_0) + \varphi \leq \frac{\pi}{2}$, then we have

$$a(t) = A\cos^n[\omega(t-t_0) + \varphi]. \quad (176)$$

If a experienced $a = 0$ and let $a|_{t=0} = 0$, then $n > 0$, $-\omega t_0 + \varphi = -\frac{\pi}{2}$, and $0 \leq \omega t \leq \pi$.

$$a(t) = A\cos^n(\omega t - \frac{\pi}{2}) = A\sin^n(\omega t). \quad (177)$$

$$\dot{a}(t) = nA\omega \sin^{n-1}(\omega t) \cos(\omega t). \quad (178)$$

Hubble parameter is

$$H \equiv \frac{\dot{a}(t)}{a(t)} = n\omega \cot(\omega t). \quad (179)$$

$$\ddot{a}(t) = nA\omega^2[(n-1)\sin^{n-2}(\omega t)\cos^2(\omega t) - \sin^n(\omega t)]. \quad (180)$$

Let us discuss the following aspects

1) If we take $a|_{t=0} = 0$, we can not take $t_0 = 0$. Otherwise the initial condition $y_0 = \frac{\dot{a}}{a}|_{t=t_0} = H|_{t=t_0} = H|_{t=0} \rightarrow \infty$ makes no sense.

2) The stage of $\ddot{a}(t) > 0$ satisfies

$$0 < \tan^2(\omega t) < n-1. \quad (181)$$

Thus $n > 1$ and $-\frac{1}{4} < \alpha < 0$ also can be obtained

3) The evolution of $a(t)$ is periodic, and the period is $T = \frac{1}{2} \times 2\pi \frac{1+2\alpha}{1+4\alpha} \sqrt{\frac{3(1+3\alpha)}{\beta}}$. In SI system, from $\beta \equiv \frac{\delta^2}{K}$ and equations (10) and (13) we can obtain.

$$T = \pi \frac{\alpha_U(1+2\alpha)}{1+4\alpha} \sqrt{\frac{3(1+3\alpha)}{4\pi G\rho_0}}. \quad (182)$$

Wherein, ρ_0 is today's inertial mass density of the universe. Let us take $\rho_0 = 0.5\% \rho_{cE0}$. $\rho_{cE0} \equiv \frac{3H_0^2}{8\pi G}$ is today's critical density of the universe mass in Einstein's theory. H_0 is today's Hubble parameter. Therefore

$$T = \frac{20\pi\alpha_U}{H_0} n \sqrt{1+3\alpha}. \quad (183)$$

4) Let t_{td} be today's value of t . t_{td} may be called the age of the universe.

$$H_0 = n\omega \cot(\omega t_{td}) = \frac{H_0}{20\alpha_U} \frac{1}{\sqrt{1+3\alpha}} \cot\left[\frac{H_0}{20n\alpha_U} \frac{t_{td}}{\sqrt{1+3\alpha}}\right]. \quad (184)$$

If t_{td} has been measured, α can be obtained from equation (184).

5) For the convenience of discussion, let $\alpha = -\frac{1}{5}$, then $n = 3$ in the following

$$a(t) = A \sin^3(\omega t). \quad (185)$$

Let us take present $\alpha_U = 0.09 \sim 0.1$ and $H_0 = 71 km.s^{-1}.Mpc^{-1}$, then

$$T \approx \frac{1}{2} \times 3284.3 \text{ hundred million years} = 1642.15 \text{ hundred million years}. \quad (186)$$

From equation (181), the conditions, which the universe of accelerating expansion satisfy, can be obtained

$$0 < \tan(\omega t) < \sqrt{2}. \quad (187)$$

From equation (184) we can obtain

$$0 < \tan(\omega t_{td}) \approx \frac{1}{2\sqrt{0.4}} < \sqrt{2}. \quad (188)$$

That is, today's universe is in a stage of accelerating expansion. From equation (180), we can also obtain $\frac{d}{dt}\ddot{a}(t) > 0$. That means today's universe expanding acceleration is increasing. From equation (184), the age of the universe can be obtained

$$t_{td} \approx 3.49674971 \times 10^{10} \text{ years} \approx 350 \text{ hundred million years}. \quad (189)$$

6) Because the evolution of $a(t)$ is periodic, there is no beginning of time. Therefore, there is no horizon problem.

7) From equations (178) and (180), we know that $\dot{a}(0) = 0$ and $\ddot{a}(0) = 0$. There is no big bang, neither is there big crunch. The Robertson-Walker metric (2) will degenerate into timelike 1-dimensional. As such, all geodesics are tangent to the geodesic of t coordinate curve at $a(t) = 0$ in the way that θ and φ are constants, and $\frac{dr}{d\beta} = 0$, $\frac{d^2r}{d\beta^2} = 0$, and $\frac{d^2t}{d\beta^2} = 0$ (β is the affine parameter of the geodesics). All geodesics are complete. That is, in the universe spacetime, there is no geodesically incomplete spacetime singularity. But still there is the divergence singularity at $a(t) = 0$. For example, $\mathfrak{R} \equiv R^{\mu\nu}R_{\mu\nu} = 9(\frac{\ddot{a}}{a})^2 + 3a^{-4}(a\ddot{a} + 2\dot{a}^2 + 2k)^2$, considering the case of $k = 0$, from equations (179), (180), and (185) we know that when $t \rightarrow 0$, $\mathfrak{R} \sim t^{-4}$ is divergent. I can hardly believe that the theory of quantum gravity could save the geodesically incomplete singularity that is formed by the collapse or the big crunch and at which the big bang will happen. I believe that the future quantum gravity theory will definitely make $a(t)$ have minimum quantum a_q ; and case $a(t) = 0$ will not happen, such that divergent singularity can be avoided.

8) Whether the universe is radiation dominated or matter dominated, the evolution of $a(t)$ is equation (177).

6.3 THE EVOLUTION OF ρ

Let Δm be the inertial mass in the co-moving volume element ΔV of the cosmic perfect fluid, then

$$\frac{1}{\Delta m} \frac{d\Delta m}{dt} = \frac{1}{a^3(t)r^2 \sin \theta \Delta r \Delta \theta \Delta \varphi \rho} \frac{d[a^3(t)r^2 \sin \theta \Delta r \Delta \theta \Delta \varphi \rho]}{dt} = \frac{1}{a^3(t)\rho} \frac{d[a^3(t)\rho]}{dt}. \quad (190)$$

Substituting equation (173) into (167), we can obtain

$$\frac{2\ddot{a}}{\beta a} + \frac{2}{3} = \frac{1}{\rho} f_1(p, \rho). \quad (191)$$

Referring to equation (30) and taking equation (190) into account, we can assume, in equation (75),

$$f_2(\rho) = -\frac{\tau}{a^3} \frac{d(a^3 \rho)}{dt} \frac{\dot{a}}{a}. \quad (192)$$

Where τ is a constant to be determined.

6.3.1 THE PERIOD OF RADIATION DOMINATED

In the radiation dominated period, $p = \frac{1}{3}\rho$, substituting equations (163) and (192) into (75), we can obtain

$$\frac{1}{\rho} f_1(p, \rho) = \frac{1}{\rho} [\hat{\beta} p + f_2(\rho)] = \frac{2}{33} - \frac{\tau}{a^3 \rho} \frac{d(a^3 \rho)}{dt} \frac{\dot{a}}{a}. \quad (193)$$

Substituting equations (185) and (193) into (191), we have

$$\frac{3\hat{\tau}}{a^3 \rho} \frac{d(a^3 \rho)}{dt} = -12\omega \cot(\omega t) - \frac{6}{11}\omega \tan(\omega t). \quad (194)$$

Where $\hat{\tau} \equiv \beta\tau$. Solving equation (194) we can obtain

$$\rho(t) = \rho_0 \left(\frac{a_0}{a}\right)^3 \left| \frac{\sin(\omega t_0)}{\sin(\omega t)} \right|^{\frac{4}{\hat{\tau}}} \left| \frac{\cos(\omega t)}{\cos(\omega t_0)} \right|^{\frac{2}{11\hat{\tau}}} = \rho_0 \left| \frac{\sin(\omega t_0)}{\sin(\omega t)} \right|^{\frac{4+9\hat{\tau}}{\hat{\tau}}} \left| \frac{\cos(\omega t)}{\cos(\omega t_0)} \right|^{\frac{2}{11\hat{\tau}}}. \quad (195)$$

Where $\rho_0 \equiv \rho(t_0)$.

6.3.2 THE PERIOD OF MATTER DOMINATED

In the matter dominated period, the pressure is $p \ll \rho$. It is negligible.

$$\frac{1}{\rho} f_1(p, \rho) \approx -\frac{\tau}{a^3 \rho} \frac{d(a^3 \rho)}{dt} \frac{\dot{a}}{a}. \quad (196)$$

Substituting equations (185) and (196) into (191), we have

$$\frac{3\hat{\tau}}{a^3\rho} \frac{d(a^3\rho)}{dt} = -12\omega \cot(\omega t) - \frac{6}{5}\omega \tan(\omega t). \quad (197)$$

Solving equation (197) we can obtain

$$\rho(t) = \rho_0 \left(\frac{a_0}{a}\right)^3 \left|\frac{\sin(\omega t_0)}{\sin(\omega t)}\right|^{\frac{4}{\hat{\tau}}} \left|\frac{\cos(\omega t)}{\cos(\omega t_0)}\right|^{\frac{2}{5\hat{\tau}}} = \rho_0 \left|\frac{\sin(\omega t_0)}{\sin(\omega t)}\right|^{\frac{4+9\hat{\tau}}{\hat{\tau}}} \left|\frac{\cos(\omega t)}{\cos(\omega t_0)}\right|^{\frac{2}{5\hat{\tau}}}. \quad (198)$$

6.3.3 THE THERMAL HISTORY OF THE UNIVERSE

Radiation satisfies the law of blackbody radiation

$$\rho = \sigma T^4. \quad (199)$$

Where, T is radiation temperature. $\sigma \equiv (\pi^2/30)N_{eff}$. N_{eff} is a constant coefficients determined by the number of particle species that the rest energy of the particles is much smaller than $k_B T$. The evolution of the radiation temperature in the radiation dominated period can be obtained from equations (195) and (199).

$$T^4(t) = \frac{\rho_0}{\sigma} \left|\frac{\sin(\omega t_0)}{\sin(\omega t)}\right|^{\frac{4+9\hat{\tau}}{\hat{\tau}}} \left|\frac{\cos(\omega t)}{\cos(\omega t_0)}\right|^{\frac{2}{11\hat{\tau}}}. \quad (200)$$

After the decoupling of Neutrino at the temperature T_ν , the ratio of neutron number n_n and proton number n_p is frozen at

$$n_n/n_p = e^{-\Delta m/T_\nu}. \quad (201)$$

Where $\Delta m \equiv m_n - m_p$ is the difference between the rest inertial masses of neutron and proton. The light split of deuterium is invalid at the temperature T_D . Let $N_\nu = 3$ be the number of types of neutrino and $Y_4 = 0.221 \sim 0.243$ be the abundance of helium, then selecting the suitable value of the ratio η of nucleon number and photon number in the radiating gas, we can determine T_ν and T_D . Let t_ν , t_D , and t_{ph} denote the time of the decoupling of Neutrino, the failure time of deuterium light split, and the time of the decoupling of photon respectively. Let the temperature of the decoupling of photon be $T_{ph} = 3000K$ and the temperature of today's photon background be $T_{td} = 2.735K$, we can obtain the following equations

$$T_\nu^4 = \frac{\rho_0}{\sigma} \left|\frac{\sin(\omega t_0)}{\sin(\omega t_\nu)}\right|^{\frac{4+9\hat{\tau}}{\hat{\tau}}} \left|\frac{\cos(\omega t_\nu)}{\cos(\omega t_0)}\right|^{\frac{2}{11\hat{\tau}}}. \quad (202)$$

$$T_D^4 = \frac{\rho_0}{\sigma} \left| \frac{\sin(\omega t_0)}{\sin(\omega t_D)} \right|^{\frac{4+9\hat{\tau}}{\hat{\tau}}} \left| \frac{\cos(\omega t_D)}{\cos(\omega t_0)} \right|^{\frac{2}{11\hat{\tau}}}. \quad (203)$$

$$T_{ph}^4 = \frac{\rho_0}{\sigma} \left| \frac{\sin(\omega t_0)}{\sin(\omega t_{ph})} \right|^{\frac{4+9\hat{\tau}}{\hat{\tau}}} \left| \frac{\cos(\omega t_{ph})}{\cos(\omega t_0)} \right|^{\frac{2}{11\hat{\tau}}}. \quad (204)$$

$$T_{td}^4 = \frac{\rho_0}{\sigma} \left| \frac{\sin(\omega t_0)}{\sin(\omega t_{td})} \right|^{\frac{4+9\hat{\tau}}{\hat{\tau}}} \left| \frac{\cos(\omega t_{td})}{\cos(\omega t_0)} \right|^{\frac{2}{11\hat{\tau}}}. \quad (205)$$

Wherein, for simplicity, the temperature of today's photon background is obtained by using the temperature equation (200) of the radiation dominated period. The inertial mass density ρ_{td} of today's universe is regarded as a known quantity - we have the following equation

$$\rho_{td} = \rho_0 \left| \frac{\sin(\omega t_0)}{\sin(\omega t_{td})} \right|^{\frac{4+9\hat{\tau}}{\hat{\tau}}} \left| \frac{\cos(\omega t_{td})}{\cos(\omega t_0)} \right|^{\frac{2}{5\hat{\tau}}}. \quad (206)$$

The 5 undetermined quantities t_ν , t_D , t_{ph} , ρ_0 , and $\hat{\tau}$ can be obtained by using the five equations (202)~(206).

As there is no big bang, the expansion of the universe is very slow in a long period of time starting from $t = 0$. There is enough time to evolve and to give the total contents of the universe evolution by the Big Bang theory. Merely the process of the universe evolution is not the same as the Big Bang theory. As a rough estimate, let us assume that the time from the start of the nucleosynthesis to today equal approximately the time of the Big Bang theory. Then the time before nucleosynthesis is approximately the same as the time $t_{\gamma d}$ of the neutrino decoupling. The time is approximately $150 \sim 200$ hundred million years. In such a long time, the standard model of particle physics maybe can give the asymmetry numbers of baryons and anti-baryons required by the formation of today's universe.

If the high-redshift of the absorption cloud in front of a quasar is the cosmological redshift, the absorption cloud is very ancient. The abundance of deuterium in the absorption cloud should be the result of primordial nucleosynthesis. The abundance of deuterium in the absorption cloud in front of all high redshift quasars should be essentially the same. Larger differences of the abundance of deuterium in different absorption cloud given by observations perhaps suggest that at least a portion of the high redshift of absorption clouds in front of quasars may not be cosmological

redshift. But both the high redshift of absorption clouds and the redshift of quasars may be the result of the inertial effects as the absorption clouds and the quasars are away from the Milky Way.

6.3.4 THE FORMATION OF THE STRUCTURE OF THE UNIVERSE

The expansion of the universe is very slow in a long period of time starting from $t = 0$. The duration of the radiation dominated period is very long. It can be seen from equation (200) that the radiation temperature is $T \sim |\sin(\omega t)|^{-\frac{4+9\tau}{4\tau}}$. If the index $\frac{4}{4\tau}$ is ignored, it is $T \sim |\sin(\omega t)|^{-\frac{9}{4}}$. For example, if in the radiation dominated period $\sin^{\frac{9}{4}}(\omega t) < 10^{-6}$, then the duration $t \sim 10^{-\frac{24}{9}} \frac{1}{\omega}$ of the radiation dominated period is about hundreds of millions of years. During either the radiation dominated period or the matter dominated period, it has enough time for the self-gravitational instability to produce respectively large-scale or small-scale mass density perturbations. The structure of the universe is forming in the manner of the top-down and bottom-up respectively.

7 THE PROBLEMS THAT MAY EXIST

At least there are two following problems for the inertial effect.

7.1 ABOUT THE INERTIAL MASS FORMULA

If the inertial mass formula (9) is correct, then, compared to the hydrogen spectrum on the Earth, blue shift will be occurred on the solar hydrogen spectrum and the pendulum period effect shown in equation (19) is easily measured; but that is not the case. Therefore, there may exist problems in the inertial mass formula (9). It is likely that equation (9) is asymptotic formula only when $r \rightarrow \infty$. In a general case, the inertial mass formula may have the following form

$$M_I \equiv M_{I12} = M_{I21} = \tilde{K} \left(\frac{r_0}{r} \right)^{n(r)} M_{G1} M_{G2} \exp(-\delta r). \quad (207)$$

Wherein, \tilde{K} and r_0 are constants which satisfy $\tilde{K}r_0 = K$. $n(r)$ is a dimensionless quantity which is dependent on r in some way, $0 \leq n(r) \leq 1$. The smaller the r is, the closer to 0 the $n(r)$ is. Below the scale of the solar system, it is $n(r) \approx 0$. The greater the r is, the closer to 1 the $n(r)$ is. Above the scale of the galactic scales, it is $n(r) \approx 1$. r_0 and $n(r)$ should be given through observation. When $n(r) = 0$, equation (207) becomes

$$M_I \equiv M_{I12} = M_{I21} = \tilde{K}M_{G1}M_{G2}\exp(-\delta r) \approx \tilde{K}M_{G1}M_{G2}. \quad (208)$$

When $r = r_0$, equation (207) becomes

$$M_I \equiv M_{I12} = M_{I21} = \tilde{K}M_{G1}M_{G2}\exp(-\delta r_0) \approx \tilde{K}M_{G1}M_{G2}. \quad (209)$$

Therefore, r_0 flags the region size of $n(r) \approx 0$. In the scale of r_0 , M_I is approximately independent to r . Then, we can let r_0 be the scale of solar system.

When the inertial mass formula (207) replaces (9), except for the pendulum effect of equation (19) and the discussions about the additional acceleration effects of the spacecraft near the Earth and the solar system in §3.1, the discussions of the rest of this article are applicable. If in the scales of solar system $n(r)$ in equation (207) is not strictly equal to 0, then the effects of pendulum period will still exist; but they are much weaker than the effect in equation (19). It can be said that the result in equation (19) is the upper limit of the effect of the pendulum period, i.e., $\frac{T_P - T_A}{T_A} \leq 10^{-4}$. As long as there is a difference between the pendulum period at perihelion and aphelion, no matter how small the difference is, it should be due to the inertial effect.

7.2 ABOUT THE "RETARDED" EFFECT

If, in vacuum, the inertial interaction propagates in the vacuum light speed c , there exists the "retarded" effect. The inertial mass formula (9) or (207) is only suitable for the static case. Because the universe is expanding, the cosmic gravitational mass density decreases with the expansion of the universe. When we calculate the contribution to the inertial mass of the testing particle by the cosmic background at

time t , the further distant the substance is, the earlier than time t the contribution to the inertial mass should be. This is the "retarded" effect.

The equation (5) is only applicable to the static universe. That is, the universe neither expands nor contracts. The slower the universe expansion is, the smaller the "retarded" effect is. Taking the attenuation factor $\exp(-\delta r)$ into account in equations (9) and (207), the further distant, i.e. the more "retarded", a substance is, the smaller the contribution to the inertial mass of the testing particle is. Therefore, the slower universe expansion is, the more reliable the result of equation (5) is. Because the gravitational mass in a volume element ε does not change with respect to t , i.e., $\frac{d}{dt}(\varepsilon \rho_G) = 0$, then $\dot{\rho}_G/\rho_G \sim \dot{\varepsilon}/\varepsilon = 3H$ can be obtained. If we think that the impact on the inertial mass of the testing particle by the substance in the distance $> 1/\delta$ can be ignored, then the relative change of the gravitational mass density in the propagation time $\frac{1/\delta}{c}$ of the inertial interaction in the distance $1/\delta$ is $(\dot{\rho}_G/\rho_G)\frac{1/\delta}{c} \sim \frac{3H}{c\delta}$. If $\frac{3H}{c\delta} \ll 1$, then we can believe that the expansion of the universe is very slow or the "retarded" effect can be ignored. Let $H = 71 \text{ km.s}^{-1}.\text{Mpc}^{-1}$ today and $1/\delta \sim \sigma r_S \sim 10^{23} \text{ m}$, then $\frac{3H}{c\delta} \sim 10^{-3}$ today. The "retarded" effect can be ignored.

Furthermore, α_U inferred by quasars is the actual value including the "retarded" effect. Due to the cosmological principle, α_U is just a function of time, independent of the spatial position. Multiplying equation (5) by the correction factor $\gamma(t)$ due to the "retarded" effect, we can obtain

$$\gamma(t)K \frac{4\pi\rho_G}{\alpha_U\delta^2} = 1. \quad (210)$$

After the universe gravitational mass density $\rho_G(t)$ is measured, $\gamma(t)$ can be determined. If the expansion of the universe is slow, $\gamma(t)$ is a slowly varying function of the time t .

The "retarded" effect is to be further studied.

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